Differential Equations

- $v = \frac{ds}{dt}$ • $a = \frac{dv}{dt}$ • $a\,ds = v\,dv$

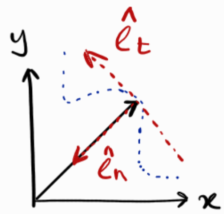
Kinematic Equations

- $v(t) = v_0 + at$ a const
- $s = s_0 + v_0 t + \frac{1}{2}at^2$ a const
- $v(s) = v^2 = v_0^2 + 2a(s - s_0)$

Projectile Motion

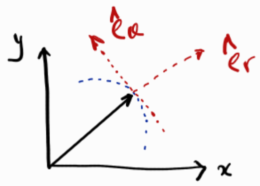
x-direction: • $v_x = \text{constant} = v_0\cos\theta$

• $x = x_0 + (v_0\cos\theta)t$ distance travelled in x



y-direction:

- $v_y = v_0\sin\theta$
- $y = y_0 + (v_0\sin\theta)t - \frac{1}{2}gt^2$ distance travelled in y
- $\vec{v}_y = v_0\sin\theta - gt$ v_y at given instant in time
- $v_y^2 = (v_0\sin\theta)^2 - 2g(y - y_0)$ v_y w.r.t. y and acceleration



Normal & Tangential

- $\vec{v} \begin{cases} v_t = r\dot{\theta} \\ v_n = 0 \end{cases}$ • $\vec{a} = \begin{cases} a_t = \dot{v} \\ a_n = \frac{v^2}{r} \end{cases}$

Circular motion:

- $\vec{a} \begin{cases} a_t = r\ddot{\theta} \\ a_n = \frac{v^2}{r} \end{cases}$ • $\vec{a} \begin{cases} a_t = r\ddot{\theta} \\ a_n = r\dot{\theta}^2 \end{cases}$

Polar Coordinates

- $\vec{v} \begin{cases} v_r = \dot{r} \\ v_\theta = r\dot{\theta} \end{cases}$ • $\vec{a} \begin{cases} a_r = \ddot{r} - r\dot{\theta}^2 \\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{cases}$

Circular Motion $(\hat{e}_r = -\hat{e}_n)$ $(\hat{e}_\theta = \hat{e}_t)$

- $\vec{v} = v\hat{e}_t = r\dot{\theta}\hat{e}_\theta$
- $\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta = \frac{v^2}{r}\hat{e}_n + \dot{v}\hat{e}_t$

Given path of motion as $f(x,y)$

$$r = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left[d^2y/d^2x\right]}$$

radius of curvature

second derivative

Constrained Motion

$y_A = -y_B$

$v_A = -v_B$

$a_A = -a_A$

method 1

$y_c = \frac{y_A + y_B}{2}$

disp. of centre

method 2

$L = y_A + y_B + C$

$0 = v_A + v_B$

Rectilinear Kinetics

$\vec{F}_g = m\vec{g}$ $\quad F_s = -k\Delta x$

static: $|\vec{F}_{static}| \leq \mu_s F_N$

kinetic: $\vec{F}_{kinetic} = \mu_k F_N$

Curvilinear Kinetics

$\Sigma F_x = ma_x$ $\quad \Sigma F_n = ma_n = m\frac{v^2}{r}$

$\Sigma F_y = ma_y$ $\quad \Sigma F_t = ma_t = m\dot{v}$

$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$

$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

Work and Energy

$$U = \int_1^2 \vec{F}\cdot d\vec{s} = \int_1^2 F\cos\theta\, ds$$

$$T_2 + V_{g_2} + V_{E_2} = T_1 + V_{g_1} + V_{E_1} + U'_{1-2}$$

Sys. Part. Work/Energy

$$\Sigma(T_2)_i = \Sigma(T_1)_i + \Sigma(U'_{1-2})_i$$

$$(T)_i = \frac{1}{2}mv_i^2$$

Linear Momentum/Impulse

$\vec{G} = m\vec{v}$ $\quad \int_1^2 \Sigma F\,dt = \vec{G} = \Delta\vec{G}$

$\vec{G}_2 = \vec{G}_1 + \int_1^2 \Sigma F\,dt$

Sys. Part. Kinetics

$\vec{r}_G = \frac{\Sigma m_i r_i}{\Sigma m_i}$ $\quad \vec{a}_G = \frac{\Sigma m_i a_i}{\Sigma m_i}$

$\vec{v}_G = \frac{\Sigma m_i v_i}{\Sigma m_i}$ $\quad \Sigma\vec{F}_i = m\vec{a}_G$

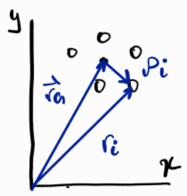
Angular Momentum/Impulse

$\vec{H}_o = \vec{r} \times m\vec{v}$ $\quad |\vec{H}_o| = |\vec{r}|m|\vec{v}|\sin\theta$

$\dot{\vec{H}}_o = \vec{r} \times \Sigma\vec{F} = \Sigma\vec{M}_o$ $\quad (\vec{H}_o)_2 = (\vec{H}_o)_1 + \int_{t_1}^{t_2} \Sigma M_o\,dt$

$\vec{v}_\theta = \vec{\omega} \times \vec{r}$ $\quad |\vec{v}_\theta| = \omega r$

Sys. Part. Linear Momentum



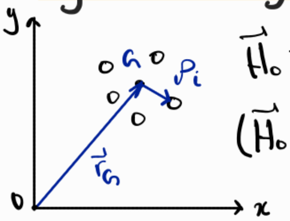
$\vec{r}_i = \vec{r}_G + \vec{\rho}_i$ $\quad \vec{G} = m\vec{v}_G$

$\vec{v}_i = \vec{v}_G + \dot{\vec{\rho}}_i \leftarrow \vec{v}_{i/G}$

$\vec{a}_i = \vec{a}_G + \ddot{\vec{\rho}}_i \leftarrow \vec{a}_{i/G}$

$$m(\vec{v}_G)_2 = m(\vec{v}_G)_1 + \Sigma\int \vec{F}_i\,dt$$

Sys. Part. Angular Momentum



$\vec{H}_O = \sum (\vec{H}_O)_i = \sum (\vec{r}_i \times m_i \vec{v}_i)$

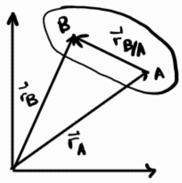
$(\vec{H}_O)_2 = (\vec{H}_O)_1 + \sum \int_{t_1}^{t_2} (M_O)_i \, dt$

$\vec{H}_G = \sum \vec{\rho}_i \times m_i \dot{\vec{\rho}}_i \leftarrow \vec{v}_{i/G}$

$\dot{\vec{H}}_G$ use product rule

$(\vec{H}_G)_2 = (\vec{H}_G)_1 + \sum \int_{t_1}^{t_2} (M_G)_i \, dt$

Relative Velocity



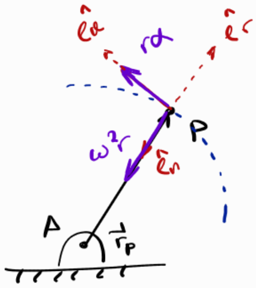
$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$|\vec{v}_{B/A}| = |\vec{\omega}||\vec{r}_{B/A}|$

$|\vec{\omega}| = \dfrac{|\vec{v}_A - \vec{v}_B|}{|\vec{r}_{A/B}|}$

$|\vec{\alpha}| = \dfrac{|\vec{a}_A - \vec{a}_B|_t}{|\vec{r}_{A/B}|}$



$\vec{a}_P = (-r\omega^2)\hat{e}_r + (r\alpha)\hat{e}_\alpha$

$\vec{a}_P = \underbrace{\vec{\omega} \times \vec{v}}_{\vec{a}_n} + \underbrace{\vec{\alpha} \times \vec{r}}_{\vec{a}_t}$

$\vec{a}_P = (-\omega^2 \vec{r}_{P/A})\hat{e}_n + (\vec{\alpha} \times \vec{r}_{P/A})\hat{e}_t$

General Plane Motion

$\sum \vec{F} = m\vec{a}_G$

$\sum \vec{M}_O = I_O \vec{\alpha}$

$\sum \vec{M}_G = I_G \vec{\alpha}$

$\sum \vec{M}_{IC} = I_{IC} \vec{\alpha}$

Radius of gyration: $I = mk^2$

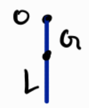
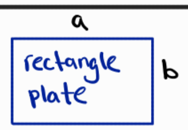
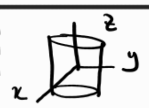
Parallel Axis Theorem:

$I_O = I_G + md^2$

No slip: $a_x = a_G = r\alpha$

correct if $F_f \le \mu_s F_N$

Yes slip: $F_f = \mu_k F_N$

				
$I_O = \frac{1}{12} mL^2$ $I_G = \frac{1}{3} mL^2$	$I_G = mR^2$	$I_G = \frac{1}{2} mR^2$	$I_G = \frac{1}{12}(a^2+b^2)$	$I_x = \frac{1}{12} m(3r^2+h^2)$ $I_z = \frac{1}{2} mr^2$

Rigid Bod. Energy

$V_g = mg\Delta h_G$

- Pure translation: $T = \frac{1}{2} m V_G^2$
- Pure rotation: $T = \frac{1}{2} I_O \omega^2 \,/\, T = \frac{1}{2} I_G \omega^2$
- Both: $T = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \omega^2$
- Couple: $U_m = \int_{\theta_1}^{\theta_2} M \, d\theta$
- if const M: $U_m = M(\theta_2 - \theta_1)$

Rigid Body Impulse / Momentum

Linear Momentum: $\vec{G} = m\vec{v}_G$ { Impulse: $\vec{G}_2 = \vec{G}_1 + \int_1^2 \sum F \, dt$

Angular Momentum about G/O: $\vec{H}_G = I_G \vec{\omega}$ or $\vec{H}_O = I_O \vec{\omega}$

Angular Momentum about A: $\vec{H}_A = I_G \vec{\omega} + m\vec{v}_G d$

Impulse: $(\vec{H}_G)_2 = (\vec{H}_G)_1 + \int_{t_1}^{t_2} \sum M_G \, dt$

Free Undamped Vibrations

standard form:

$\ddot{x} + \omega_n^2 x = 0$

solutions:

$x = A\cos\omega_n t + B\sin\omega_n t$

$x = C\sin(\omega_n t + \phi)$

parameters:

$x_0 = A = C\sin\phi$

$\dot{x}_0 = B\omega_n = C\omega_n \sin\phi$

$C = \sqrt{A^2 + B^2}$

$\phi = \arctan\left(\frac{B}{A}\right)$

natural frequency [rad/s]: $\omega_n = \sqrt{\frac{k}{m}}$

period (s): $T = \dfrac{2\pi}{\omega_n}$

natural frequency [Hz]:

$f_n = \dfrac{1}{T} = \dfrac{\omega_n}{2\pi}$

Free Damped Vibrations

standard form:

$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 0$

$\zeta = \dfrac{c}{2m\omega_n} = \dfrac{c}{c_c}$

solutions:

$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

$\lambda = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$

Overdamped ($\zeta > 1$, $c > c_c$)

- roots λ are distinct, real, <0

Critical damped ($\zeta = 1$, $c = c_c$)

$\lambda_1 = \lambda_2 = -\omega_n$

solution: $x = (A_1 + A_2 t)e^{-\omega_n t}$

Underdamped ($\zeta < 1$, $c < c_c$)

- roots λ distinct, imaginary

solution: $x = C\sin(\omega_d t + \phi)e^{-\zeta\omega_n t}$

$\omega_d = \omega_n\sqrt{1 - \zeta^2}$ $\quad T_d = \dfrac{2\pi}{\omega_d}$

Rigid Body Vibrations

replace x — θ

$\dot{x}$ — $\dot{\theta}$

$\ddot{x}$ — $\ddot{\theta}$

m — I_O

For particles: $\sum F_x = ma_x$

For rigid bodies:

$\sum M_O = I_O \alpha$ $\quad \sin\theta \approx \theta$

$\sum M_G = I_G \alpha$ $\quad \cos\theta \approx 1$

Basic Stuff

$\dfrac{\sin A}{\alpha} = \dfrac{\sin B}{\beta} = \dfrac{\sin C}{c}$

$c^2 = a^2 + b^2 - 2ab\cos C$