$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\delta(t)$	1
$\delta(t-a)$	e^{-as}

$f(t)$	$F(s)$
$\cos(kt)$	$\frac{s}{s^2+k^2}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$
$e^{at}\cos(kt)$	$\frac{s-a}{(s-a)^2+k^2}$
$e^{at}\sin(kt)$	$\frac{k}{(s-a)^2+k^2}$

Time Domain	Laplace Domain
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)\cdots-f^{(n-1)}(0)$
$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$
$t^nf(t)$	$(-1)^n\frac{d^n}{ds^n}F(s)$
$e^{at}f(t)$	$F(s-a)$
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$g(t)u(t-a)$	$e^{-as}\mathcal{L}\{g(t+a)\}$

$\sin 2\theta = \sin\theta\cos\theta$

$2\cos^2\theta - 1 = \cos 2\theta$

$1-2\sin^2\theta = \cos 2\theta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Residues: $s=p$ is pole order n : $\rightarrow \frac{1}{(n-1)!}\left[\frac{d^{n-1}}{ds^{n-1}}\left(G(s)\cdot(s-p)^n\right)\right]_{s=p}$

$G(s) \triangleq e^{st}F(s) \Rightarrow f(t) = \sum_{i=1}^{N} \mathrm{Res}(G(s), s=p_i)$

$n=1: \mathrm{Res}(G(s), s=p) = \left[G(s)(s-p)\right]_{s=p}$

$n=2: \mathrm{Res}(G(s), s=p) = \frac{d}{ds}\left[G(s)\cdot(s-p)^2\right]_{s=p}$

MIE stuff

$\sum F = ma$

$\sum M = I\ddot{\theta}$

$I_O = I_G + md^2$

Disk: $I_G = \frac{1}{2}mR^2$

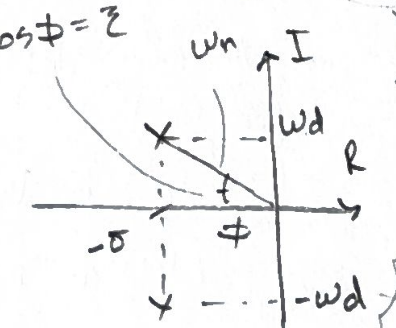
Ring: $I_G = mR^2$

Stick: $I_O = \frac{1}{12}ml^2$, $I_G = \frac{1}{3}ml^2$

$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}$ → piecewise cont's → exponentially bounded

$\cos t = \frac{e^{jt}+e^{-jt}}{2}$ $\sin t = \frac{e^{jt}-e^{-jt}}{2j}$

$\cos\phi = \zeta$



$G(s) = C(sI-A)^{-1}B + D$

$i_c = C\frac{dv_c}{dt}$, $V_L = L\frac{di_L}{dt}$

$Y(s) = \frac{b}{s^2+as+b}$, $s = -\frac{a}{2} \pm j\frac{\sqrt{4b-a^2}}{2}$

$Y(s) = \frac{\sigma^2+\omega_d^2}{(s+\sigma)^2+\omega_d^2}$, $s = -\sigma \pm j\omega_d$

$Y(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$, $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$\sigma = \zeta\omega_n$ $\zeta = \frac{\sigma}{\sqrt{\sigma^2+\omega_d^2}}$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$ $\omega_n = \sqrt{\sigma^2+\omega_d^2}$

$\dot{x} = Ax + Bu$, $y = Cx + Du$ $\Rightarrow$ $\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$, $\tilde{y} = C\tilde{x} + D\tilde{u}$

$A = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & & \frac{\partial \dot{x}_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial \dot{x}_m}{\partial x_1} & \cdots & \frac{\partial \dot{x}_m}{\partial x_n} \end{bmatrix}$, $B = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial u} \\ \vdots \\ \frac{\partial \dot{x}_m}{\partial u} \end{bmatrix}$

$x = \bar{x} + \tilde{x}$

$y = \bar{y} + \tilde{y}$

$u = \bar{u} + \tilde{u}$

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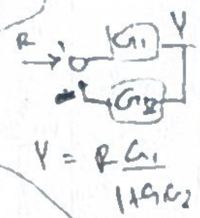
$$A\cos(\omega t) + B\sin(\omega t) = C\sin(\omega t + \phi)$$

$$C = \sqrt{A^2 + B^2} \qquad \phi = \arctan\left(\frac{B}{A}\right)$$



$Y = R\frac{G_1}{1+G_1G_2}$

$\zeta > 1$: overdamped

$\zeta = 1$: critically damped

$0 < \zeta < 1$: underdamped

Routh Array: checking if roots of a polynomial are in OLHP:

$$a(s) = k_1 s^5 + k_2 s^4 + k_3 s^3 + k_4 s^2 + k_5 s + k_6$$

s^5	k_1	k_3	k_5	0	0
s^4	k_2	k_4	k_6	0	0
s^3	a_1	a_2	0	0	0
s^2	b_1	b_2	0	0	0
s^1	c_1	0	0	0	0
s^0	0				

If sign change in 1st col = means unstable (or reach 0 by s^0)

$$a_1 = -\frac{1}{k_2}\det\begin{bmatrix} k_1 & k_3 \\ k_2 & k_4 \end{bmatrix} \qquad b_2 = -\frac{1}{a_1}\det\begin{bmatrix} k_2 & k_6 \\ a_1 & 0 \end{bmatrix}$$

$$a_2 = -\frac{1}{k_2}\det\begin{bmatrix} k_1 & k_5 \\ k_2 & k_6 \end{bmatrix} \qquad c_1 = \frac{1}{b_1}\det\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$$

$$b_1 = -\frac{1}{a_1}\det\begin{bmatrix} k_2 & k_4 \\ a_1 & a_2 \end{bmatrix}$$

Final Value Thm

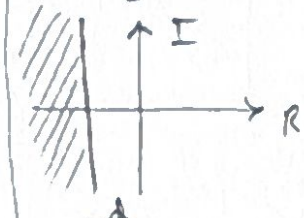
$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

FV DNE if lim DNE or

1) RHP poles
2) >1 pole at origin.

Control Specs: TF with 2 complex conjugate poles and no zeroes

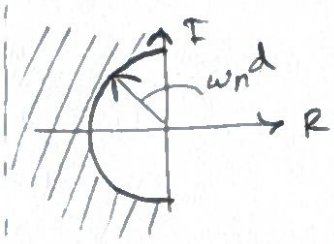
$$T_s = \frac{4}{\zeta\omega_n}$$



$\sigma \geq \frac{4}{T_s^d} = \sigma^d$

for $T_s \leq T_s^d$

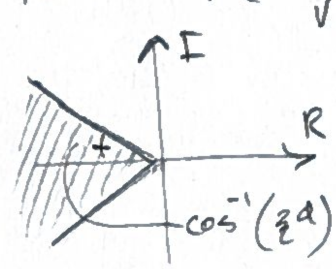
$$T_r = \frac{1.8}{\omega_n}$$



$\omega_n \geq \frac{1.8}{T_r^d} = \omega_n^d$

for $T_r \leq T_r^d$

$$\%OS = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$



$\zeta \geq \zeta^d$ for $\%OS \leq \%OS^d$

$$\zeta^d = \frac{-\ln(\%OS^d)}{\sqrt{\pi^2 + \ln^2(\%OS^d)}}$$

OLS TF: $U \rightarrow \boxed{G(s)} \rightarrow Y$

Thm 1: Asymptotic Stab: Asymptotically stable iff roots of "$\det(sI-A)=0$" are in OLHP, or poles of $X_i(s)$ rows are in OLHP

Thm 2: BIBO Stab: BIBO stable iff all poles of $G(s)$ are in OLHP → use Routh to check

CLS is BIBO stable iff (Thm 1):

1) poles of $\frac{1}{1+C(s)G(s)}$ in OLHP
2) no unstable pole-zero cancellation in the product $C(s)G(s)$

(a)

$s^2 + as + b$ iff $a, b > 0$, then real part <0 for roots (in OLHP)

polynomial order $k-1 \xrightarrow{\mathcal{L}} \frac{N(s)}{s^k}$: k poles at origin

Assuming BIBO stab,
(b) **Asymptotic Tracking** iff:

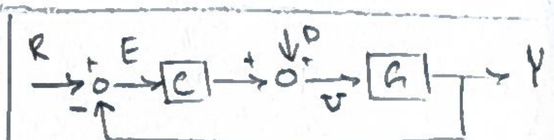
- $C(s)G(s)$ has k poles at $s=0$ ($r(t)$ is polynomial order $k-1$)

↳ if type $k-1$, non-zero finite error

↳ if type $k-2$ or less, $e(\infty)$ blows up

Assuming BIBO stab,
(c) **Disturbance Rejection** iff:

- $C(s)$ is type j (j poles @ $s=0$) ($d(t)$ is polynomial order $j-1$)
- poles of $G(s)$ won't help with disturbance rejection.



$$E(s) = \frac{1}{1+CG}R + \frac{-G}{1+CG}D$$

$$U(s) = \frac{C}{1+CG}R + \frac{1}{1+CG}D$$

Internal Model Principle: $C(s)$ solves tracking problem iff:

1) $C(s)$ makes CLS BIBO stable
2) the product $C(s)G(s)$ contains the poles of $R(s)$
3) $C(s)$ contains the poles of $D(s)$

Basic Control Problem unsolvable if any zeroes of $G(s)$ are poles of $R(s)$

asymptotic stability implies BIBO stab.

For control specs, holds if extra pole(s) real part is further left (5-10x) compared to your dominant poles. LHP zero ok if very diff, if RHP, BAD → non minimum phase.