$u(t) \to \frac{1}{s}$ $\quad \cos(kt) \to \frac{s}{s^2+k^2}$ $\quad f^{(n)}(t) \to s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$

$t \to \frac{1}{s^2}$ $\quad \sin(kt) \to \frac{k}{s^2+k^2}$ $\quad \int_0^t f(\tau)d\tau \to \frac{F(s)}{s}$

$t^2 \to \frac{2!}{s^3}$ $\quad e^{at}\cos(kt) \to \frac{s-a}{(s-a)^2+k^2}$ $\quad \int_0^t f(\tau)g(t-\tau)d\tau \to F(s)G(s)$

$t^n \to \frac{n!}{s^{n+1}}$ $\quad e^{at}\sin(kt) \to \frac{k}{(s-a)^2+k^2}$ $\quad t^n f(t) \to (-1)^n \frac{d^n}{ds^n}F(s)$

$e^{at} \to \frac{1}{s-a}$ $\quad f(t-a)u(t-a) \to e^{-as}F(s)$

$\delta(t) \to 1$ $\quad e^{at}f(t) \to F(s-a)$

$\delta(t-a) \to e^{-as}$ $\quad g(t)u(t-a) \to e^{-as}\mathcal{L}\{g(t+a)\}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin 2\theta = \sin\theta\cos\theta$

$2\cos^2\theta - 1 = \cos 2\theta = 1 - 2\sin^2\theta$

$\Sigma F = ma$ $\quad I_o = I_G + md^2$ $\quad$ Ring: $I_G = mR^2$

$\Sigma M = I\ddot{\theta}$ $\quad$ disk: $I_G = \frac{1}{2}mR^2$ $\quad$ stick: $I_o = \frac{1}{12}ml^2$, $I_G = \frac{1}{3}ml^2$

Residues: $s=p$ is pole order n $\to \frac{1}{(n-1)!}\left[\frac{d^{n-1}}{ds^{n-1}}\left(G(s)\cdot(s-p)^n\right)\right]_{s=p}$

$G(s) \triangleq e^{st}F(s) \to f(t) = \sum_{i=1}^{M} \mathrm{Res}(G(s), s=p_i)$

$n=1$: $\mathrm{Res}(G(s), s=p) = \left[G(s)(s-p)\right]_{s=p}$

$n=2$: $\mathrm{Res}(G(s), s=p) = \frac{d}{ds}\left[G(s)\cdot(s-p)^2\right]_{s=p}$

$\cos t = \frac{e^{jt}+e^{-jt}}{2}$, $\sin t = \frac{e^{jt}-e^{-jt}}{2j}$

$\dot{x} = Ax + Bu$ $\quad x = \bar{x} + \tilde{x}$

$y = Cx + Du$ $\quad y = \bar{y} + \tilde{y}$

$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u}$ $\quad u = \bar{u} + \tilde{u}$

$\tilde{y} = C\tilde{x} + D\tilde{u}$

$$A = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \cdots & \frac{\partial \dot{x}_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial \dot{x}_m}{\partial x_1} & \cdots & \frac{\partial \dot{x}_m}{\partial x_n} \end{bmatrix} \quad B = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial u} \\ \vdots \\ \frac{\partial \dot{x}_m}{\partial u} \end{bmatrix}$$

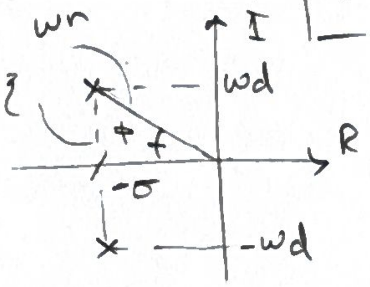
$G(s) = C(sI-A)^{-1}B + D$

$Y(s) = \frac{b}{s^2+as+b}$, $s = -\frac{a}{2} \pm j\frac{\sqrt{4b-a^2}}{2}$

$Y(s) = \frac{\sigma^2+\omega_d^2}{(s+\sigma)^2+\omega_d^2}$, $s = -\sigma \pm j\omega_d$

$Y(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$, $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$\cos\phi = \zeta$



$\sigma = \zeta\omega_n$

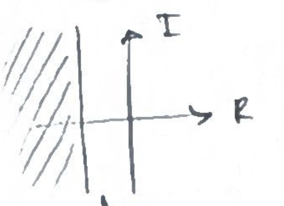
$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\zeta = \frac{\sigma}{\sqrt{\sigma^2+\omega_d^2}}$

$\omega_n = \sqrt{\sigma^2+\omega_d^2}$

Control Specs: TF with 2 complex conjugate poles and no zeroes

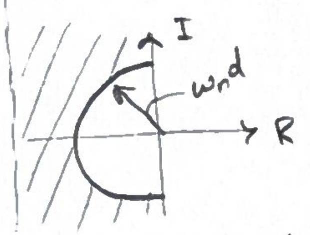
$T_s = \frac{4}{\zeta\omega_n}$



$\sigma \geq \frac{4}{T_s^d} = \sigma^d$

for $T_s \leq T_s^d$

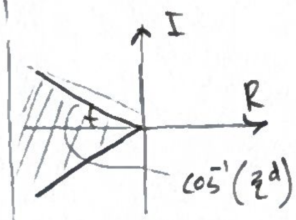
$T_r = \frac{1.8}{\omega_n}$



$\omega_n \geq \frac{1.8}{T_r^d} = \omega_n^d$

for $T_r \leq T_r^d$

$\%OS = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)$



$\zeta \geq \zeta^d$ for $\%OS \leq \%OS^d$

$\zeta^d = \frac{-\ln(\%OS^d)}{\sqrt{\pi^2+\ln^2(\%OS^d)}}$

OLS TF: $u \to \boxed{G(s)} \to y$

Thm1: Asymptotic Stab: Asymptotically stable iff roots of $\det(sI-A)=0$ are in OLHP, or poles of $X_i(s)$ rows are in OLHP.

Thm2: BIBO stab. BIBO stable iff all poles of $G(s)$ are in OLHP → use Routh to chk

Asymp. stab. implies BIBO stab.

$s^2 + as + b$: iff $a,b>0$, then OLHP (roots have $\mathrm{Re}<0$)

polynomial order $k-1$ $\xrightarrow{\mathcal{L}}$ $\frac{N(s)}{s^k}$: k poles @ origin

CLS is (a) BIBO stable iff (Thm1)

1) poles of $\frac{1}{1+CG}$ are in OLHP
2) no unstable pole-zero cancellation in the product $C(s)G(s)$

Assuming (a), (b) Asymptotic Tracking iff

- $C(s)G(s)$ has k poles at $s=0$ ($r(t)$ is polynomial order $k-1$)

↳ if type $k-1$, non-zero finite error

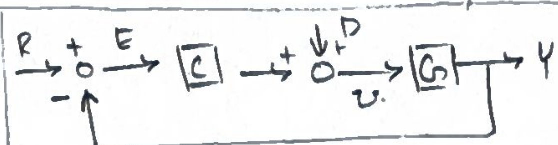
↳ if type $k-2$ or less, $e(\infty)$ blows up

Assuming (a), (c) Disturbance Rejec^r iff

- $C(s)$ is type j (j poles at $s=0$) ($d(t)$ is polynomial order $j-1$)
- poles of $G(s)$ won't help with disturbance rejection

IMP: $C(s)$ solve tracking problem iff:

1) $C(s)$ makes CLS BIBO stable
2) the product $C(s)G(s)$ contains the poles of $R(s)$
3) $C(s)$ contains the poles of $D(s)$



$E(s) = \frac{1}{1+CG}R + \frac{-G}{1+CG}D$

$u(s) = \frac{C}{1+CG}R + \frac{1}{1+CG}D$

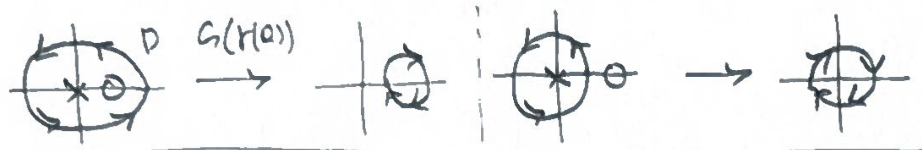
BCP unsolvable if any zeroes of $G(s)$ are poles of $R(s)$

FVT: $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

FV DNE if lim DNE or

1) RHP 2) >1 pole at origin

Argument Principle: Parametrize a CCW contour D with $s = \gamma(\theta)$ to created transformed contour $G(\gamma(\theta))$. $G(\gamma(\theta))$ encircles origin $(Z - P)$ times CCW, where Z = # zeroes of $G(s)$ and P = # poles of $G(s)$ inside D



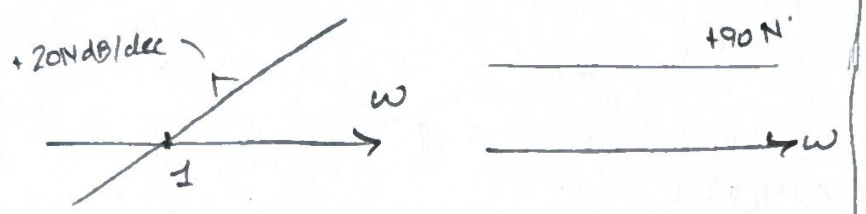
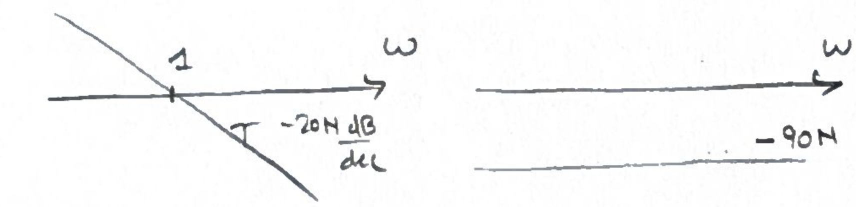
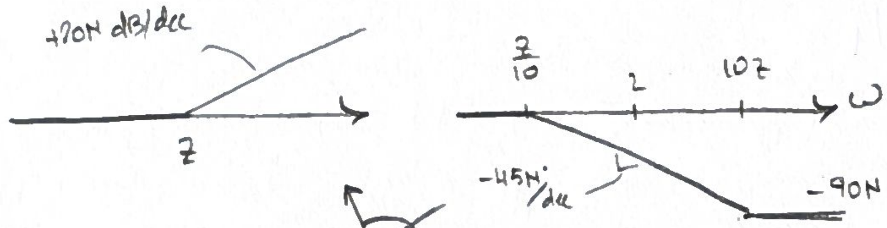
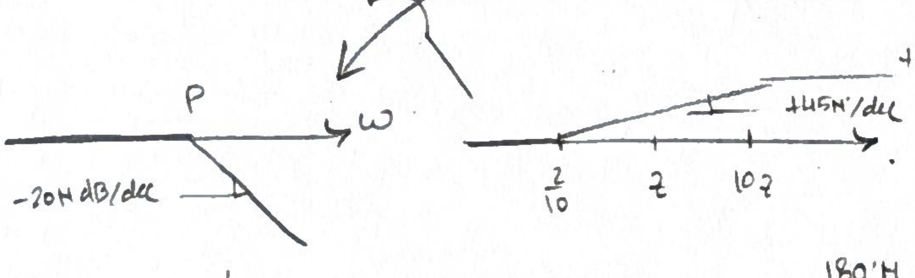
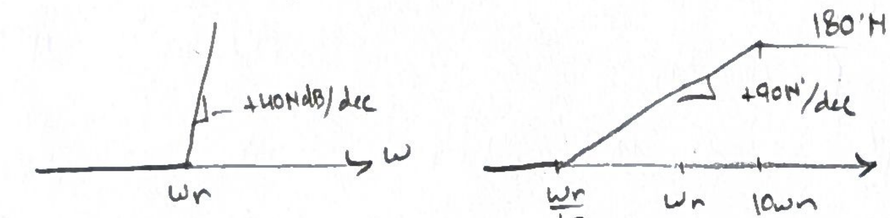
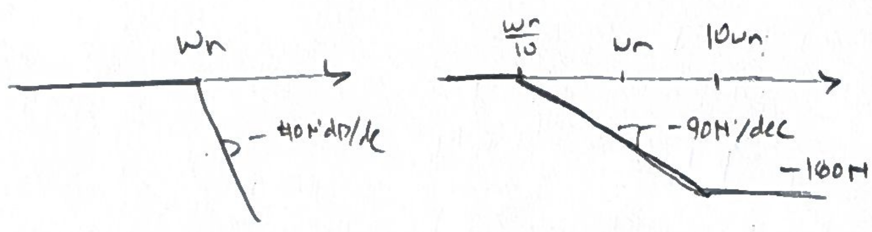
Nyquist Criterion: Our D is RHP going CW:

$$\begin{pmatrix}\#\ L \text{ encircles} \\ \omega = -1/k \text{ CW}\end{pmatrix} = -\begin{pmatrix}\#\text{ poles of} \\ L(s) \text{ inside D}\end{pmatrix}$$

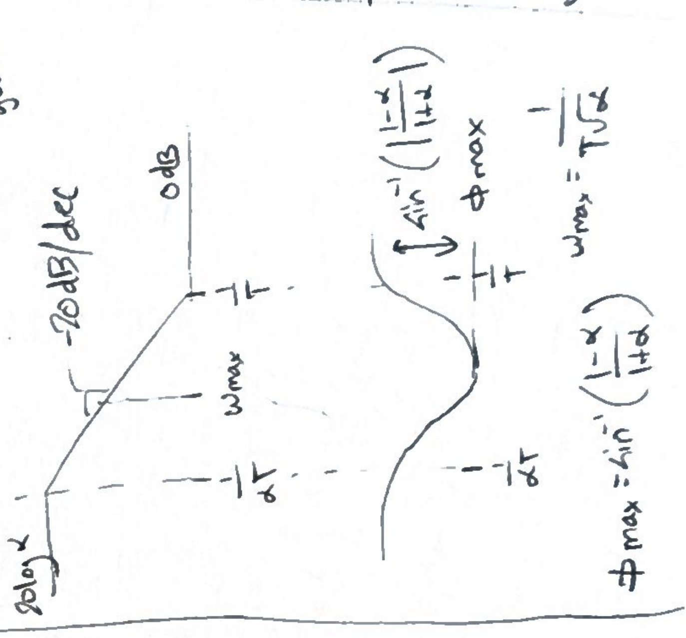
$\#\ L$ encircles $\omega = -1$ CCW $=$ # poles of $L(s)$ insid D

Roots of $1 + KL(s)$ are in OLHP iff:
1) L doesn't cross pnt $-\frac{1}{k}$
2) L encircles $-\frac{1}{k}$ p times CCW, p = # poles of $L(s)$ in RHP

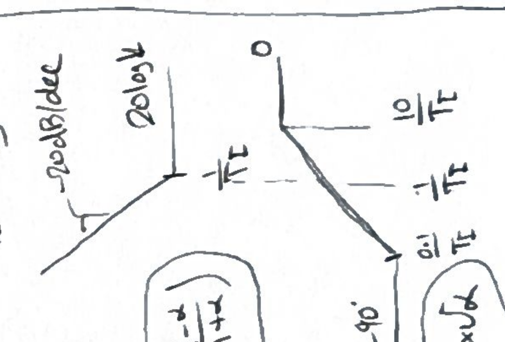
indentation of contours, L can blow up for D $R \to \infty$, L collapses to origin.

factor	magnitude	phase
K		
$(s)^N$		
$\frac{1}{(s)^N}$		
$\left(1 + \frac{s}{z}\right)^N$		
$\frac{1}{\left(1 + \frac{s}{p}\right)^N}$		
$\left(1 + \frac{2\zeta s}{\omega_n} + \left(\frac{s}{\omega_n}\right)^2\right)^N$		
$\frac{1}{\left(1 + \frac{2\zeta s}{\omega_n} + \left(\frac{s}{\omega_n}\right)^2\right)^N}$		

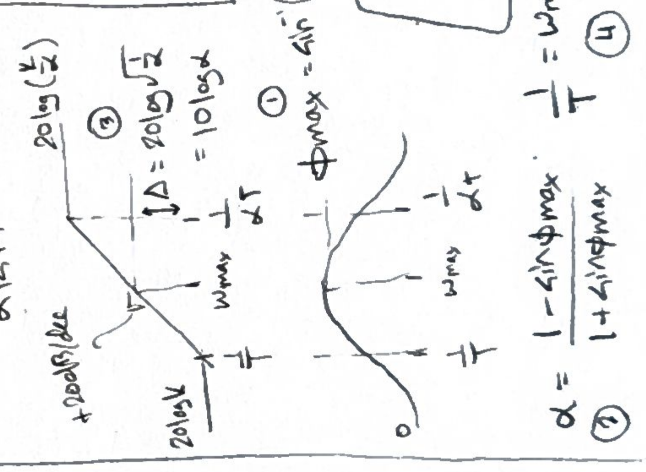
Lag Controller $C(s) = \alpha \frac{Ts+1}{\alpha Ts + 1}$, $T > 0$, $\alpha > 1$ — amplify DC gain



PI Controller (lag controller type) $C(s) = K\left(1 + \frac{1}{T_I s}\right) = \frac{K}{T_I} \cdot \frac{T_I s + 1}{s}$

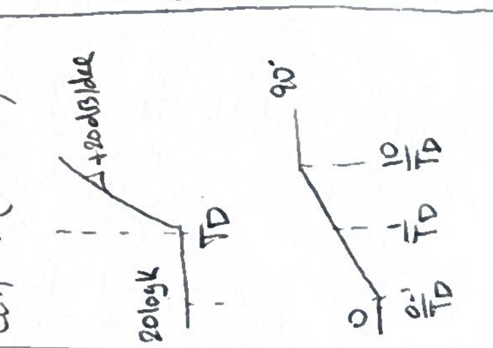


Lead Controller (inc PM) $C(s) = K \frac{Ts+1}{\alpha Ts + 1}$, $K, T > 0$, $0 < \alpha < 1$



① $\phi_{max} = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$ ② $\alpha = \frac{1 - \sin\phi_{max}}{1 + \sin\phi_{max}}$ ③ $\Delta = 20\log\sqrt{\frac{1}{\alpha}} = 10\log\alpha$ ④ $\frac{1}{T\sqrt{\alpha}} = \omega_{max}$

PD Controller (inc PM, ω_c) $C(s) = K(1 + T_D s)$



PD: ① increase PM by placing $\frac{1}{T_D}$ b4 ω_c ② used to inc. ω_c.

Lead: ① incr. phase at ω_{max} if prop designed ② get inc. PM → %OS ↓ ③ byproduct if ω_c ↑, then T_s ↓

Steps: ① set $K=1$ or to meet ss error. ② α from desired PM. ③ place ω_{max} at crossover ④ find Δ thru $10\log\alpha$ and see where it intercepts the origina Bode, and that's ω_{max} → ⑤ find $\frac{1}{T}$.

PI: ① choose K to meet tracking and bandwidth ② find crossover freq of $K \cdot G(j\omega)$, call it ω_c (suppose we happy ω_c, PM) ③ set $\frac{1}{T_I} \ll \omega_c$, $\frac{1}{T_I}$ b/n $0.01\omega_c$ and $0.1\omega_c$

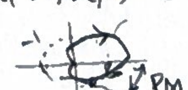
Steps: ① choose α for desired amplification of DC gain ② : choose $\frac{1}{T}$ b/n $0.01\omega_c$ and $0.1\omega_c$ ③ : $C(0) = \alpha$, $e(\infty) = -\frac{1}{\alpha}$

Lag: ① amplify DC gain for better tracking and dist. reject.

$\omega_c \approx \frac{1}{2}\omega_{BW}$

PM>0 : suggest no encirclement of −1
PM<0 : suggest yes encircle of −1

$PM = \measuredangle L(j\omega) - (-\pi)$



$$PM = \operatorname{atan}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}\right)$$

$$\omega_{BW} = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$GM = \frac{1}{|L(j\bar{\omega})|}$

want $\frac{1}{GM} < 1 \to GM > 1$

$K < GM \to$ BIBO since $\frac{1}{K} > \frac{1}{GM}$