Unit symbol: a f p n μ m c d k M G T P E

$10^\wedge$: −18 −15 −12 −9 −6 −3 −2 −1 3 6 9 12 15 18

Examination Aid Sheet

Faculty of Applied Science & Engineering

Both sides of the sheet may be used;
must be printed on 8.5" x 11" paper.

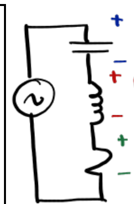
$P = I^2 R = \frac{V^2}{R} = IV$

Subject: ECE212 : Circuit Analysis

Candidate's name:_________________________

Candidate's signature:_________________________

Series RLC



BP: HP↔LP

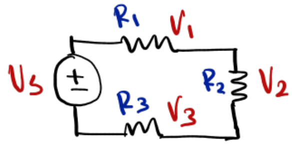
BS: HP//LP

Ckt Pwr: (+) current enters

(+) terminal : pwr dissapated, > 0

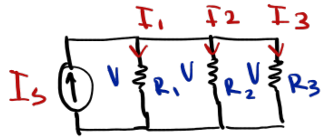
(−) terminal : pwr supplied, < 0

Voltage Divider:



$V_k = V_s \cdot \frac{R_k}{\sum_{j=1}^{N} R_j}$

Current Divider:

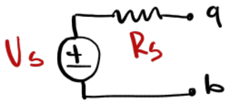


$I_k = I_s \cdot \frac{\frac{1}{R_k}}{\sum_{j=1}^{N} \frac{1}{R_j}}$

Nodal Analysis: Use KCL to find Voltage at Nodes, $N - 1 - N_V = \#$ equ.

Mesh Analysis: Use KVL to find Current through Loops, $B - N + 1 - N_I = \#$ equ.

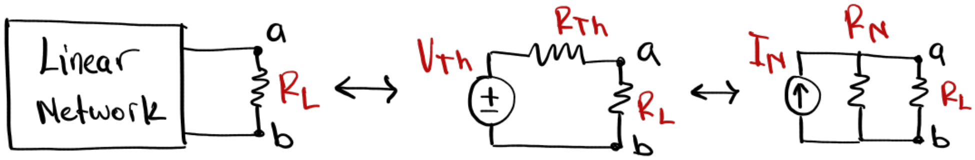
Source Transformation



↔

$\boxed{V_s = I_s R_p}$ $\boxed{R_p = R_s}$

Thevenin and Norton Ckts:



$\boxed{V_{Th} = I_N R_N}$

$\boxed{R_{Th} = \frac{V_{Th}}{I_{sc}} = R_N}$

Shortcut (only indep. srces): deactivate all srces, R_{Th} = equivalent resistance from terminal of interest

Condition for Max Power ($R_L = R_{Th}$): $P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{V_{Th}^2}{4R_L}$

First Order Transient ODE: $\tau \frac{dx(t)}{dt} + x(t) = x(\infty)$

solution: $\underbrace{K_1}_{x(\infty)} + \underbrace{K_2}_{x(0^+) - x(\infty)} e^{-t/\tau}$

$$\boxed{x(t) = x(\infty) + [x(t_0^+) - x(\infty)] e^{\frac{-(t-t_0)}{\tau}}}$$

capacitors: $\tau = R_{Th} C_{equ}$ and $V_C(t_0^-) = V_C(t_0^+)$

inductors: $\tau = \frac{L_{equ}}{R_{Th}}$ and $i_L(t_0^-) = i_L(t_0^+)$

Second Order Transient complete solution: $x(t) = x_{natural}(t) + x_{forced}(t) = x_n(t) + x(\infty)$

Series RLC: $i'' + \frac{R}{L} i' + \frac{1}{LC} i = 0$

Parallel RLC: $V'' + \frac{1}{RC} V' + \frac{1}{LC} V = 0$

$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$

$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

ζ < 1 : underdamped; = 1 : critically damped; > 1 : overdamped

AC Ckt Analysis $x(t) = A\cos(\omega t + \phi) \longrightarrow \bar{X} = Ae^{j\phi} = A\angle\phi = A(\cos\phi + j\sin\phi)$

1. must have same frequency
2. all cos or all sin
3. peak amplitude same sign
4. difference < 180° or π rad

add 2π or 360°

$\sin(\omega t + \phi) = \cos(\omega t + \phi - \frac{\pi}{2})$

$\cos(\omega t + \phi) = \sin(\omega t + \phi + \frac{\pi}{2})$

$-\sin(\omega t + \phi) = \sin(\omega t + \phi - \pi)$

$-\cos(\omega t + \phi) = \cos(\omega t + \phi + \pi)$

$Z_R = R$, V and I in phase

$Z_L = j\omega L$, V leads I by 90°

$Z_C = \frac{1}{j\omega C}$, I leads V by 90

Resistors $V_R = I_R R$

Series: $R_T = R_1 + R_2 + R_3$

Parallel: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Capacitor $i_C = CV_C'$

Series: $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

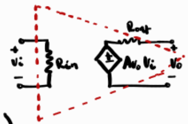
Parallel: $C_T = C_1 + C_2 + C_3$

Inductors $V_L = L i_L'$

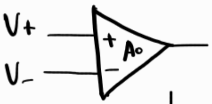
Series: $L_T = L_1 + L_2 + L_3$

Parallel: $\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

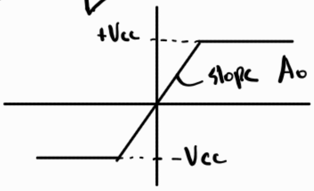
Op Amps



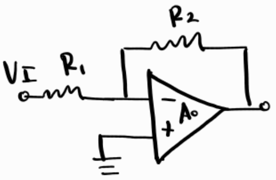
Ideal:
1) $i_+ = i_- = 0$ (opamp)
2) $V_- = V_+$ (negative feedback)



$V_o = A_o(V_+ - V_-)$

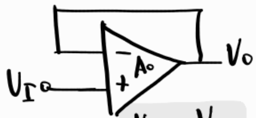


Inverting Config:



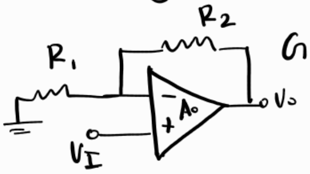
$$G = \frac{V_o}{V_I} = \frac{-\frac{R_2}{R_1}}{1 + \frac{(1+R_2/R_1)}{A_o}} \leftarrow K$$

Unity Gain Buffer



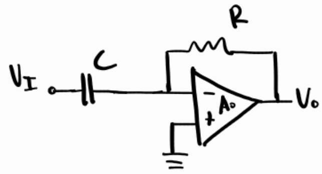
$V_o = V_I$

Noninverting Config:



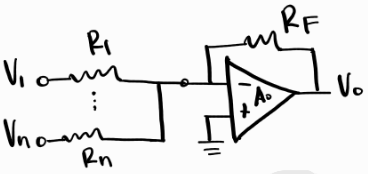
$$G = \frac{V_o}{V_I} = \frac{1 + \left(\frac{R_2}{R_1}\right)}{1 + \frac{(1+R_2/R_1)}{A_o}} \leftarrow K$$

Differentiator



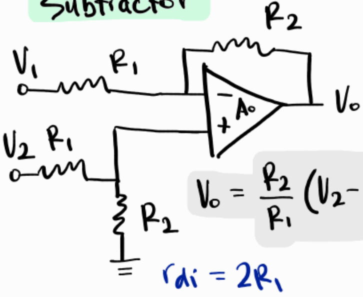
$V_o = -RC \frac{d}{dt} V_I$

Weighted Sum Amplifier



$V_o = -R_F\left(\frac{V_1}{R_1} + \cdots + \frac{V_n}{R_n}\right)$

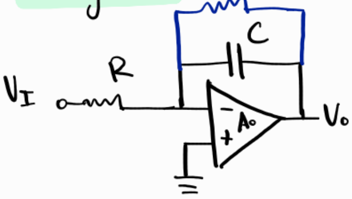
Subtractor



$V_o = \frac{R_2}{R_1}(V_2 - V_1)$

$r_{di} = 2R_1$

Integrator



$V_o = -\frac{1}{RC}\int V_I \, dt$

Inductors/Transformers

$L = \frac{N\Phi}{i} = \frac{\mu_o N^2 A}{l}$

$\Phi = B \cdot A = \mu_o \frac{N}{l} i A$

$M \le \sqrt{L_1 L_2}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

Turns Ratio: $n = \frac{N_2}{N_1}$ **Voltage Ratio:** $\frac{V_2(t)}{V_1(t)} = \pm \frac{N_2}{N_1} = \pm n$ (+ additive, − subtractive) **Current Ratio:** $\frac{i_2(t)}{i_1(t)} = \mp \frac{1}{n}$

Reflected Impedence: $Z_{in} = \frac{Z_L}{n^2}$ $Z_L = n^2 Z_s$

Dot Convention: additive if both currents entering/leaving dots, subtractive otherwise

Power

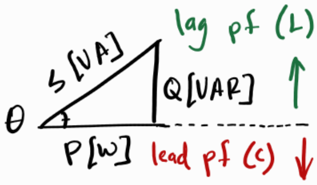
$P_{avg} = R I_{rms}^2 = V_{rms}^2/R$, **Thevenin Max Pwr** if $Z_L = Z_{TH}^*$, $P_m = [V_{Th\,rms}]^2/4R_{th}$ $V_{pk} = \sqrt{2} V_{rms}$

$S \triangleq \bar{V}_{rms} \bar{I}_{rms}^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$

$= V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$= P + jQ = I_{rms}^2 Z = V_{rms}^2 / Z^* = V_{rms}^2 Y^*$

$\underbrace{P_{avg}}_{\text{real pwr [W]}} = \overbrace{V_{rms} I_{rms}}^{\text{apparent pwr [VA]}} \underbrace{\cos(\overbrace{\theta_v - \theta_i}^{\text{pf angle}})}_{pf = \frac{P}{S}}$

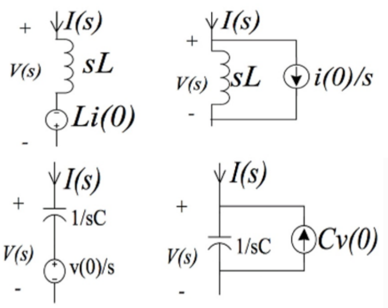


$f(t)$	$F(s)$
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\delta(t-a)$	e^{-as}

$f(t)$	$F(s)$
$\cos kt$	$\frac{s}{s^2+k^2}$
$\sin kt$	$\frac{k}{s^2+k^2}$
$\cosh kt$	$\frac{s}{s^2-k^2}$
$\sinh kt$	$\frac{k}{s^2-k^2}$
$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$
$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$

Time Domain	s-Domain
$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$
$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$\int_0^t f(\tau) g(t-\tau) d\tau$	$F(s)G(s)$
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$
$e^{at} f(t)$	$F(s-a)$
$f(t-a)u(t-a)$	$e^{-as} F(s)$
$g(t)u(t-a)$	$e^{-as} \mathcal{L}\{g(t+a)\}$
$f(t) = f(t+T)$	$\frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$

For complex roots, $F(s) = \cdots + \frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta}$
where $k = |k|\angle\theta$ and $k^* = |k|\angle-\theta$
the inverse Laplace:
$f(t) = \left(\cdots + 2|k| e^{-\alpha t} \cos(\beta t + \theta) + \cdots\right) u(t)$



$T(s)$ transfer function	Filter type
$\frac{K}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$	low-pass
$\frac{K(s/\omega_0)^2}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$	high-pass
$\frac{K(s/\omega_0)}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$	bandpass
$\frac{K[(s/\omega_0)^2 + 1]}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1}$	bandstop