Unit symbol: a f p n μ m c d K M G T P E
10 ^ : −18 −15 −12 −9 −6 −3 −2 −1 3 6 9 12 15 18

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1 q_2}{r^2}$$ Electric force at distance r.

$$|\vec{E}_q| = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}$$ Magnitude of $\vec{E}$ at distance r from q

$$\Phi_E = \oiint_S \vec{E}\cdot d\vec{A} = \frac{Q_{enc}^*}{\epsilon_0}$$ Gauss Law

Biot-Savart Law $d\vec{B} = \frac{\mu_0 i\, d\vec{s}\times\hat{r}}{4\pi r^2}$

$\vec{F}_Q = \vec{E}Q^*$ Force on charge Q in electric field $\vec{E}$

$$\oiint_S \vec{E}\cdot d\vec{A} = E\cdot\text{area}\cdot\cos\angle\vec{E}\cdot\hat{n} = \frac{Q_{enc}^*}{\epsilon_0}$$

Assuming $\vec{E}$ uniform and angle const.

Charge Densities

$$|\vec{E}| = \frac{\sigma}{2\epsilon_0}$$ Electric field for non-conducting sheet of charge

$$|\vec{E}| = \frac{\lambda}{2\pi\epsilon_0 r}$$ Electric field for line of charge at distance r

$$|\vec{E}| = \frac{\sigma}{\epsilon_0}$$ Electric field for charged plate

Seri − Q
Par − V

Capacitance $\left[\frac{C}{V}\right] = [F]$

$$C = \frac{Q}{V} \qquad V = Ed$$

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$ parallel plate capacitor

$$C = \frac{Q}{V} = \frac{2\pi\epsilon_0 h}{\ln\left(\frac{b}{a}\right)}$$ cylindrical capacitor height h, outer radius b and inner radius a

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_0 ab}{b-a}$$ spherical capacitor inner a, outer b

$$C = 4\pi\epsilon_0 R$$ isolated sperical capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}qV$$ potential energy in capacitor

$$u = \frac{1}{2}\epsilon_0 E^2 = \left[\frac{J}{m^3}\right]$$ energy density stored in electric field

Magnetic Field $[T] = \left[\frac{V\cdot s}{m^2}\right] = \left[\frac{Wb}{m^2}\right]$

$$\oiint \vec{B}\cdot d\vec{A} = 0$$ Gauss Law

$$[\mu_0] = \left[\frac{V\cdot s}{A}\cdot\frac{1}{m}\right] = \left[\frac{H}{m}\right]$$

$$\vec{F}_B = q\vec{v}\times\vec{B}, \qquad \vec{F}_B = I\vec{L}\times\vec{B}$$

$$B = \frac{\mu_0 I}{2\pi R}$$ B at dist. R from wire

$$B = \frac{\mu_0 I \Phi}{4\pi R}$$ dist. R angle Φ in radian

$$F_{b,a} = \frac{\mu_0 L I_a I_b}{2\pi d}$$ F on wire b due to a at distance d b.t. them

parallel attract
opposite repel

$$\oint_c \vec{B}\cdot d\vec{A} = \mu_0 I_{enc}$$ Ampere's Law

$$B = \mu_0 n I$$ B in solenoid

$$B = \left(\frac{\mu_0 i}{2\pi R}\right) r$$ B inside wire

A N = total # of loops
n = # of loops /unit length
$n = N/\ell$, $N = n\ell$

ℓ

Potential Energy

$$U_F - U_i = \Delta U = W_{app} = -W_{field}$$

$$U = \frac{q_1^* q_2^*}{4\pi\epsilon_0 r}$$ electric potential energy of 2 particle system

Voltage $\left[\frac{J}{C}\right] = [V]$

$$V_F - V_i = \Delta V = \frac{\Delta U}{q^*} = \frac{W_{app}}{q^*} = -\frac{W_{field}}{q^*}$$

$$\Delta V = -\int_i^F \vec{E}\cdot d\vec{s}$$ change in voltage as we move through $\vec{E}$

$$V_q(r) = \frac{q^*}{4\pi\epsilon_0 r}$$ Voltage of point charge q at distance r from charge

Resistors Circuits

Series: $R_T = R_1 + R_2 + R_3$

Parallel: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$+\ V_R\ -$, R, $\rightarrow I_R$

$V_R = I_R R$

Capacitor Circuits

I_C, $+\ V_C\ -$

Series: $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Parallel: $C_T = C_1 + C_2 + C_3$

$$I_c = C\frac{dV_c}{dt}$$

No current open to DC

Inductors Circuits

I_L, $+\ V_L\ -$

no Voltage short to DC

Series: $L_T = L_1 + L_2 + L_3$

Parallel: $\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$

$$V_L = L\frac{dI}{dt}$$

Inductance $[H] = \left[\frac{V\cdot s}{A}\right]$

$$\mathcal{E}_{EMF} = -N\frac{d}{dt}\Phi_B = -N\frac{d}{dt}\iint \vec{B}\cdot d\vec{A}$$

opposing $\Delta\Phi_B$

time derivative of Φ_B

$$L = \frac{N\Phi_B}{I} = n^2 A\ell\mu_0$$ Inductance for inductor, cross.s.area A

$$\mathcal{E}_{\substack{self\\induced}} = \mathcal{E}_L = -\frac{d}{dt}N\Phi_B$$ self induction, opposes $\Delta\Phi_B$ in itself.

$$V_L = -\mathcal{E}_L = L\frac{dI}{dt}$$ Voltage across inductor with inductance L

$$U = \frac{1}{2}LI^2$$ energy stored in inductor with inductance L and current I

$$\frac{dW}{dt} = P = \vec{F}\cdot\frac{d\vec{x}}{dt} = \vec{F}\cdot\vec{v}$$

Δwork, power, velocity

$$u = \frac{B^2}{2\mu_0}$$ Energy density of B field $[J/m^3]$

Resistance/Current

$[\frac{V}{A}] = \Omega$ $\quad i = \int \vec{J}\cdot d\vec{A}$ $\quad \vec{J} = (ne)\vec{v}_d$ $\quad q = (nAL)e$

n: # of charge carriers by volume
A: cross sec. wire
L: length wire
e – fundamental charge

$\vec{J} = [\frac{A}{m^2}]$ current density $\quad I = JA$

$\rho = \frac{E}{J} = [\Omega\cdot m]$ resistivity

$\sigma = [\frac{1}{\Omega\cdot m}]$ conductivity $\quad \sigma\vec{E} = \vec{J}$

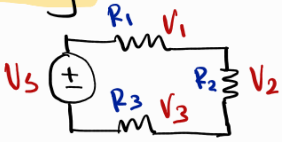
$R = \rho\frac{L}{A}$ resistance of wire length L, resistivity ρ, and area A

$P = I^2R = \frac{V^2}{R} = IV$ Power dissapation $P = [W] = [A\cdot V]$

Ckt Pwr: (+) current enters (+) terminal: pwr dissapated > 0
(−) terminal: pwr supplied < 0

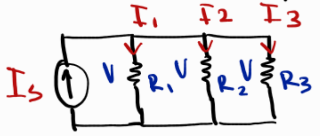
Circuit Theory

Voltage Divider:



$V_k = V_s \cdot \frac{R_k}{\sum_{j=1}^{N} R_j}$

Current Divider:

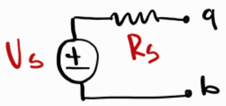


$I_k = I_s \cdot \frac{\frac{1}{R_k}}{\sum_{j=1}^{N} \frac{1}{R_j}}$

Nodal Analysis: Use KCL to find Voltage at Nodes, $N - 1 - N_V = \#$ equ.

Mesh Analysis: Use KVL to find Current through Loops, $B - N + 1 - N_I = \#$ equ.

Source Transformation



$\leftrightarrow$

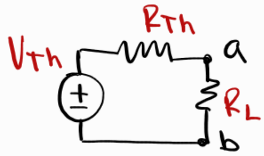
$\boxed{V_s = I_sR_p}$ $\quad \boxed{R_p = R_s}$

Superposition: Find desired V or I by deactivating one (or more) srces at a time. (Sum up)

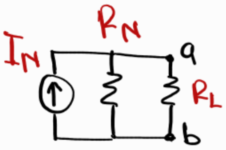
Linearity & Scaling: $V_{out} = \alpha_1 V_{s1} + \alpha_2 V_{s2} + \dots + \beta_1 I_{s1} + \beta_2 I_{s2}$ When αs and βs known from previous calculations, new V_{out} with new V_s and I_s can be calculated.

Thevenin and Norton Ckts:

$\leftrightarrow$



$\leftrightarrow$



$\boxed{V_{Th} = I_NR_N}$

$\boxed{R_{Th} = \frac{V_{Th}}{I_{sc}} = R_N}$

Shortcut (only indep. srces): deactivate all srces, R_{Th} = equivalent resistance from terminal of interest

Condition for Max Power ($R_L = R_{Th}$): $P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{V_{Th}^2}{4R_L}$

First Order Transient

ODE: $\tau\frac{dx(t)}{dt} + x(t) = x(\infty)$

solution: $\underbrace{K_1}_{x(\infty)} + \underbrace{K_2}_{x(0^+) - x(\infty)} e^{-t/\tau}$

$$\boxed{x(t) = x(\infty) + [x(t_0^+) - x(\infty)]e^{\frac{-(t-t_0)}{\tau}}}$$

capacitors: $\tau = R_{Th}C_{equ}$ and $V_C(t_0^-) = V_C(t_0^+)$

inductors: $\tau = \frac{L_{equ}}{R_{Th}}$ and $i_L(t_0^-) = i_L(t_0^+)$

AC Ckt Analysis

$x(t) = A\cos(\omega t + \phi)$

A: peak amplitude
ω: angular frequency [rad/s]
ϕ: phase angle [rad] or [deg]

1. must have same frequency
2. all cos or all sin
   - $\sin(\omega t + \phi) = \cos(\omega t + \phi - \frac{\pi}{2})$
   - $\cos(\omega t + \phi) = \sin(\omega t + \phi + \frac{\pi}{2})$
3. peak amplitude same sign
   - $-\sin(\omega t + \phi) = \sin(\omega t + \phi - \pi)$
   - $-\cos(\omega t + \phi) = \cos(\omega t + \phi + \pi)$
4. difference $< 180^\circ$ or π rad
   add 2π or 360°

R: $R \to R$, V and I in phase: $\underline{V} = R\underline{I}$

L: $L \to j\omega L$, V leads I by 90°: $\underline{V} = j\omega L\underline{I}$

C: $C \to \frac{1}{j\omega C} = \frac{-j}{\omega C}$, I leads V by 90°: $\underline{V} = \frac{1}{j\omega C}\underline{I}$

$$\boxed{Z = x + jb = Ae^{j\theta} = A\angle\theta = A\cos\theta + jA\sin\theta}$$

Eg: given $V(t) = V_m\cos(\omega t + \theta_v)$
$\Rightarrow V(t) = Re[\underline{V}e^{j\omega t}]$ where
$\Rightarrow \underline{V} = V_m\angle\theta_v$ phasor $\underline{V}$ doesn't depend on t

$Z = R + jX \to Z$: impedance, R: resistance, X: reactance

$Y = \frac{1}{Z} = G + jB \to Y$: admittance, G: conductance, B: susceptance

$G = \frac{R}{R^2 + X^2}$ $\quad R = \frac{G}{G^2 + B^2}$

$B = \frac{-X}{R^2 + X^2}$ $\quad X = \frac{-B}{G^2 + B^2}$

Constants

$k = 8.99\times10^{9}\ [\frac{N\cdot m^2}{C^2}]$ $\quad$ Sphere $V = \frac{4}{3}\pi r^3$ $\quad SA = 4\pi r^2$

$\epsilon_0 = 8.85\times10^{-12}\ [\frac{F}{m}]$ $\quad \mu_0 = 4\pi\times10^{-7}\ [\frac{H}{m}]$ $\quad -j = 1\angle{-\pi/2} = \frac{1}{j}$

$e = q_e = 1.602\times10^{-19}\ [C]$ $\quad 1eV = 1.602\times10^{-19}\ [J]$ $\quad -1 = 1\angle{\pm\pi} = j\cdot j$