Introduction | Coulomb's Law | Lecture 1 ~ Week 0

→ Electromagnetic Forces

$\vec{F} = m\vec{a}$ $\quad\quad$ $\vec{F}_E$: electric force

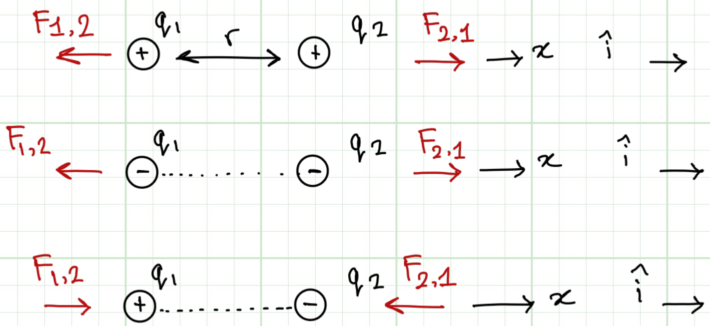
$\vec{F}_{EM}$: electromagnetic force $\quad\quad$ $\vec{F}_B$: magnetic force

→ Charges

1) charge is conserved $\quad$ $e^- + e^+ = \gamma + \gamma$ ← photons have zero charge
2) charge is quantized (1 coulomb : $\pm 1.6 \times 10^{-19}$ C)

→ Coulomb's Law

* like charges repel
* unlike charges attract
* force acts along line joining charges
*



Notes

* $F_{2,1}$ is the force on charge 2 by charge 1.
* $|\vec{F}_{2,1}| = |\vec{F}_{1,2}| = F_{1,2}$ ... no arrow

$$|\vec{F}_{1,2}| = |\vec{F}_{2,1}| = k \frac{|q_1||q_2|}{r^2}$$

$k = 9 \times 10^9$

$$k = \frac{1}{4\pi\varepsilon_0}$$

ε_0 = permitivity of free space or vacuum $= 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$

Units

$q_1 = [C]$

$$|\vec{F}_{1,2}| = k \frac{|q_1||q_2|}{r^2}$$

$r = [m]$

$F = [N]$

$k = \left[\frac{N\,m^2}{C^2}\right]$

$$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

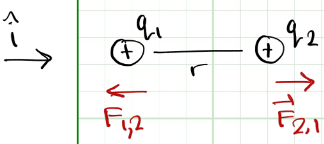
$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{m^2 N} = \frac{\text{Farad}}{\text{metre}} = \frac{F}{m}$

$\uparrow$ permitivity of free space

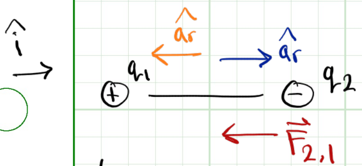
$F(x,y,z) = x^2\hat{i} + 2xy\hat{j} + z\hat{k}$ $\quad F: \mathbb{R}^3 \to \mathbb{R}^3$

$$\left.\begin{aligned} f(x,y,z) &= x^2+y^2+z^2 \\ f(1,1,1) &= 3 \end{aligned}\right\} \text{scalar function}$$

* Magnitude and Direction



$$\left.\begin{aligned} \vec{F}_{2,1} &= \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}\hat{i} \\ \vec{F}_{1,2} &= -\frac{|q_1||q_2|}{4\pi\varepsilon_0 r^2}\hat{i} \end{aligned}\right\} \text{Physical Argument}$$



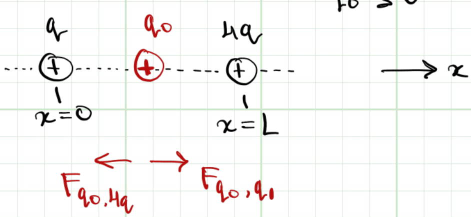
Vectoral argument

$$\vec{F}_{2,1} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{a_r}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1|q_2|}{r^2}\hat{i}$$

$\hat{a_r}$ is a unit vector that points from source to the destination

Example



$q_0 \lessgtr 0$

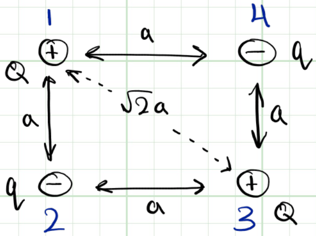
Find location of q_0 s.t. $F_{net} = 0$

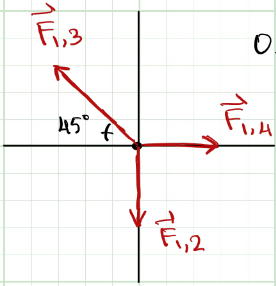
- we know q_0 must be in the middle somewhere
- we know q_0 should be closer to q because $4q$ has more force

$$\left.\begin{aligned} |\vec{F}_{q_0,q}| &= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q q_0}{x^2} \\ |\vec{F}_{q_0,4q}| &= \frac{1}{4\pi\varepsilon_0}\cdot\frac{4q q_0}{(L-x)^2} \end{aligned}\right\} \frac{q q_0}{x^2} = \frac{4q q_0}{(L-x)^2} \Rightarrow \boxed{x = L/3}$$

Example 2

Square system : Find $\frac{Q}{|q|}$ s.t. $\vec{F}_1{}_{net} = 0$





$$0 = \vec{F}_{1,2} + \vec{F}_{1,3} + \vec{F}_{1,4}$$

$$F_{1,3} = k\frac{Q^2}{2a^2}$$

$\cos 45°$

$$\boxed{\vec{F}_{1,3} = -\frac{kQ^2}{2a^2}\frac{1}{\sqrt{2}}\hat{i} + \frac{kQ^2}{2a^2}\frac{1}{\sqrt{2}}\hat{j}}$$

$$\boxed{\vec{F}_{1,2} = -\frac{kQ|q|}{a^2}\hat{j}} \qquad \boxed{\vec{F}_{1,4} = \frac{kQ|q|}{a^2}\hat{i}}$$

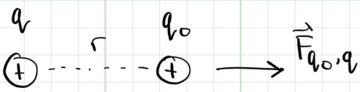
$$\vec{F}_{1net} = \left[-\frac{kQ^2}{2a^2}\cdot\frac{1}{\sqrt{2}} + \frac{kQ|q|}{a^2}\right]\hat{i} + \left[\frac{kQ^2}{2a^2}\cdot\frac{1}{\sqrt{2}} - \frac{kQ|q|}{a^2}\right]\hat{j}$$

Set components equal to 0 and solve for $\frac{Q}{|q|}$

$$\boxed{\therefore \frac{Q}{|q|} = 2\sqrt{2}}$$

Electric Field

Week 0: Lec 3 — Electric Field



$$\boxed{\vec{E}_q = \frac{\vec{F}_{q_0,q}}{q_0}}$$

Assumptions

- q_0 is positive
- q_0 is so small that it does not affect q's field

* field ~ charge or modification of space

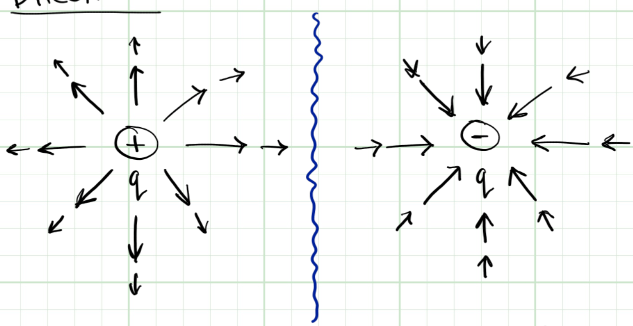
$$\vec{E}_q = \lim_{q_0 \to 0} \frac{\vec{F}_{q_0,q}}{q_0} \quad \Big\} \text{ limit def of } \vec{E}$$

$$|\vec{E}_q| = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{|q||\cancel{q_0}|}{r^2}}{|\cancel{q_0}|} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q|}{r^2}$$

magnitude of electric field at distance r from q

$$\boxed{|\vec{E}_q| = \frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2}}$$

Direction



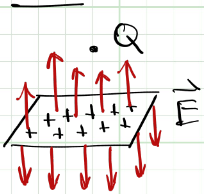
$$|\vec{E}_q| = \frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2}$$

$$\vec{E}_q = \frac{\vec{F}_{q_0 q}}{q_0}$$

$$\boxed{\vec{F}_{q_0 q} = q_0 \vec{E}_q}$$

Electric field lines always start from positive charges and terminate at negative charges

Force



$$\vec{F}_Q = \vec{E}\, Q$$ ← has an algebraic sign

$\vec{E}$

$\oplus\ Q \longrightarrow \quad \vec{F}_Q = \vec{E}\,Q$ (same direction as $\vec{E}$)

$\ominus\ Q \longleftarrow \quad \vec{F}_Q = \vec{E}(-Q)$ (opposite direction as $\vec{E}$)

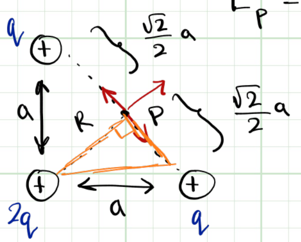
* A positive charge moves along $\vec{E}$
+ A negative charge moves against $\vec{E}$

Units for Electric Field

$$\vec{E} = \left[\frac{N}{C}\right] = \left[\frac{\text{Volts}}{\text{metre}}\right] = \left[\frac{V}{m}\right]$$

Example 1:

$\vec{E}_P = ?$



Superposition:

$$\vec{E}_T = \cancelto{0}{\vec{E}_q + \vec{E}_q} + \vec{E}_{2q}$$

$$|\vec{E}_{2q}| = \frac{1}{4\pi\epsilon_0}\frac{2q}{R^2}$$

$$\vec{E}_{2q} = |\vec{E}_{2q}|\cos 45^\circ\,\hat{i} + |\vec{E}_{2q}|\sin 45^\circ\,\hat{j}$$

$$R^2 + \left(\frac{\sqrt{2}}{2}a\right)^2 = a^2$$

$$R^2 + \frac{2}{4}a^2 = a^2$$

$$R^2 = \frac{a^2}{2}$$

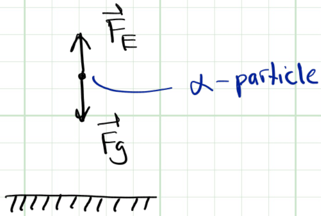
$$\boxed{\therefore \vec{E}_{2q} = \frac{2q}{4\pi\epsilon_0}\frac{a^2}{2}\left[\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]}$$

Example 2

$m_\alpha = 6.64 \times 10^{-27}$ kg

charge of $\alpha = 2e$

$e = 1.602 \times 10^{-19}$ C



$$\vec{F}_E = 2q\vec{E} = mg = \vec{F}_g$$

$$2q\vec{E} = mg$$

$$\vec{E} = \frac{mg}{2q}$$

Gauss Law (electric field)

- is a subset of Maxwell Equations

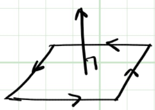
$$\underbrace{\overbrace{\oint\!\!\!\oint_S \vec{E} \cdot d\vec{A}}^{\text{electric flux}}}_{\text{surface integral over a closed surface}} = \frac{Q_{enc}}{\varepsilon_0}$$

enclosed charges (algebraic sign)

differential surface element

encloses a volume

Differential Surface Element

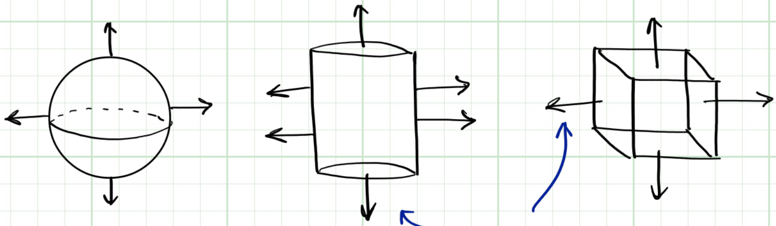


$$\vec{A} = A\hat{n}$$

unit vector normal to plane

follows RHR

magnitude



normal unit vectors alway outward

$$\vec{dA} = dA\,\hat{n}$$

normal unit vectors

tiny surface

Double Integral

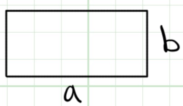
$$\int_{y_1}^{y_2}\int_{x_1}^{x_2} f(x)g(y)\,dx\,dy = \int_{y_1}^{y_2} g(y)\,dy \int_{x_1}^{x_2} f(x)\,dx = G(y)\Big|_{y_1}^{y_2} F(x)\Big|_{x_1}^{x_2}$$

Example $$\int_{y=0}^{y=1}\int_{x=1}^{x=2} 3xy^2\,dx\,dy = 3\int_{y=0}^{y=1} y^2\,dy \int_{x=1}^{x=2} x\,dx$$

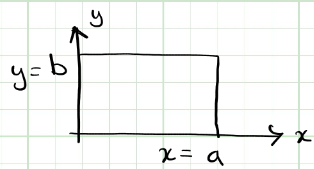
$$= 3\left(\tfrac{1}{3}y^3\right)\Big|_0^1 \left(\frac{x^2}{2}\right)\Big|_1^2$$

$$= 3\left(\tfrac{1}{3}\left(\frac{2^2}{2} - \frac{1}{2}\right)\right)$$

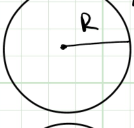
$$= 3\left(\frac{1}{3}\times\frac{3}{2}\right) = 3\left(\frac{3}{6}\right) = \boxed{\frac{3}{2}}$$



Area = ab

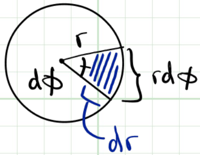


$$\int_{y=0}^{y=b}\int_{x=0}^{x=a} dx\,dy = \int_{y=0}^{y=b} dy \int_{x=0}^{x=a} dx = a\int_{y=0}^{y=1} dy = ab$$



Area = πr^2

$$\int_{r=0}^{r=R}\int_{\phi=0}^{\phi=2\pi} \underbrace{r\,dr\,d\phi}_{dA} = \int_{r=0}^{R} r\,dr\,[\phi]_0^{2\pi}$$



$$= 2\pi\int_{r=0}^{R} r\,dr = 2\pi\left(\frac{r^2}{2}\right)_0^R = \pi R^2$$

Surface Integral Trick

$$\oint\!\!\oint_S \vec{E}\cdot d\vec{A} = \oint\!\!\oint |\vec{E}||d\vec{A}|\cos\angle \vec{E}\cdot d\vec{A}$$

$$= \oint\!\!\oint E\,dA\cos\angle \vec{E}\cdot\hat{n}$$

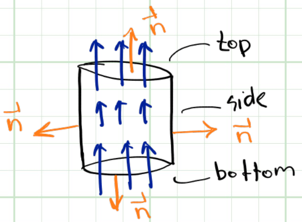
assuming $\vec{E}$ uniform and angle remains the same.

$$= E\cos\angle E\cdot\hat{n}\boxed{\iint d\vec{A}} = \frac{Q_{en}}{\varepsilon_0}$$

$$= E\cdot \text{area}\cdot\cos\angle E\cdot\hat{n} = \frac{Q_{enc}}{\varepsilon_0}$$

Example

$\Phi_E = ?$



Electric flux $\Phi_E = 0$ because whatever's going in goes out.

$$\oint\!\!\oint \vec{E}\cdot d\vec{A} = \iint_{\text{bottom}} \vec{E}\cdot d\vec{A} + \iint_{\text{top}} \vec{E}\cdot d\vec{A} + \iint_{\text{side}} \vec{E}\cdot d\vec{A}$$

$$= \iint_{\text{bottom}} E\,dA\cos\angle E\cdot\hat{n} + \iint_{\text{top}} E\,dA\cos\angle E\cdot\hat{n} + \iint_{\text{bottom}} E\,dA\cos\angle E\cdot\hat{n}$$

$$= E(-1)\,dA + E(1)\,dA + E(0)\,dA$$

$$= E(-1)\,A_{bot} + E(1)\,A_{top} + E(0) \quad = 0$$

Week 1: Lec 2

Charge Density

- Linear charge distribution : $Q = \int \lambda \, ds$ $[C/m]$ — diff. length
- Surface charge distribution : $Q = \iint \sigma \, dA$ $[C/m^2]$ — diff. area
- Volume charge distribution : $Q = \iiint \rho \, dv$ $[C/m^3]$ — diff volume

Application of Gauss Law

$\oplus \overset{r}{\text{———}}$ $E_q = \dfrac{kq}{r^2} = \dfrac{|q|}{4\pi\varepsilon_0 r^2}$

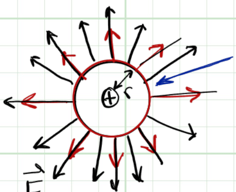
$$\iint_{GS} \vec{E} \cdot \vec{dA} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\iint_{GS} E \cos\angle \vec{E}, \hat{n} \, dA = \frac{Q_{enc}}{\varepsilon_0}$$

choose Gaussian surface wisely s.t. E and angle bt. E and $\hat{n}$ are constant.

↳ can take out of integral

- We can say point charge is symmetric sphere
- Therefore $\vec{E}$ must be radially symmetric



$\vec{E}$

Gaussian surface, $\vec{E}$ is uniform and angle between $\vec{E}$ and $\hat{n}$ is same

$$\oiint \vec{E} \cdot \vec{dA} = \oiint E \underbrace{\cos\angle \vec{E}, \hat{n}}_{\cos(0) = 1} \, dA = E \oiint dA = \frac{Q}{\varepsilon_0}$$

$$E \times \text{Surface Area} = \frac{Q}{\varepsilon_0}$$

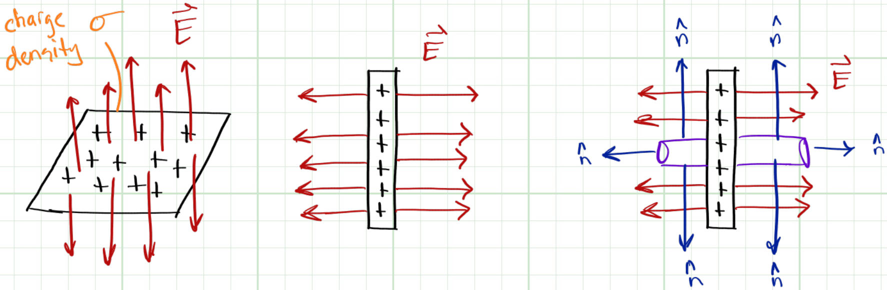
$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$F_q = \vec{E} Q \Rightarrow \vec{E} = \frac{F}{Q}$$

$$\frac{F}{Q} = \frac{Q}{4\pi\varepsilon_0 r^2} \Rightarrow \boxed{F = \frac{Q^2}{4\pi\varepsilon_0 r^2}}$$

Example: Surface Charge Density



$$\oiint \vec{E}\cdot d\vec{A} = \underset{\text{left}}{\oiint} \vec{E}\cdot d\vec{A} + \underset{\text{right}}{\oiint} \vec{E}\cdot d\vec{A} + \underset{\text{side}}{\oiint} \vec{E}\cdot dA$$

$$= \underset{\text{left}}{\oiint} E\cos(0)\, dA + \underset{\text{right}}{\oiint} E\cos(0)\, dA + \underset{\text{side}}{\oiint} E\,\cancel{\cos(1)}^{\,0}\, dA$$

$$= 2E \oiint dA$$

$$= 2EA$$

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

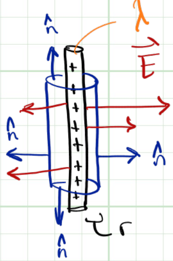
$$2EA = \frac{Q}{\varepsilon_0}$$

$$2E\cancel{A} = \frac{\cancel{A}\sigma}{\varepsilon_0}$$

$$\boxed{|\vec{E}| = E = \frac{\sigma}{2\varepsilon_0}}$$

electric field for charged surface (sheet of charge)

Example : Line Surface density



$$\oiint \vec{E}\cdot d\vec{A} = \iint_{top} \vec{E}\cdot d\vec{A} + \iint_{bott} \vec{E}\cdot d\vec{A} + \iint_{side} \vec{E}\cdot d\vec{A}$$

$$= \iint_{top} E\cos(90)\,dA + \iint_{bott} E\cos(90)\,dA + \iint_{side} E\cos(0)\,dA$$

$$= E\iint_{side} dA = EA = \underline{\underline{2\pi r h E}}$$

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$2\pi r h E = \frac{Q}{\epsilon_0}$$

$$2\pi r \cancel{h} E = \frac{\lambda \cancel{h}}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r \epsilon_0}$$

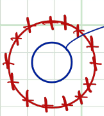
$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

electric field for charged line

First Shell Theorem

- a charged particle inside a charged shell experiences no force from the charged shell.
- this means that there is zero electric field inside an empty charged shell.

Proof

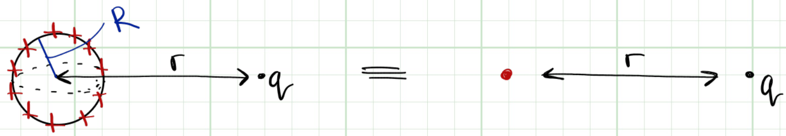


$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon}$$

we know $Q=0$ because the shell is empty, which means eighter area is zero or electric field is zero or angle b.t. $\hat{n}$ and $\vec{E}$ is always 90°

$\therefore$ Only way LHS=RHS is if $\vec{E}$ is 0

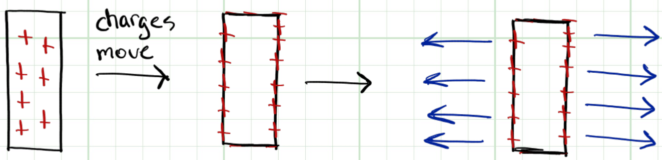
Second Shell Theorem



- you can take all the amount of charge of the shell and concentrate it at the centre.
- replace shell with point charge $Q = 4\pi R^2 \sigma$ at centre

Charged Plate

- Any charge put inside the perfect metal moves to the surface
- $\vec{E}$ inside perfect metal is zero.



$$\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}}$$

$\vec{E}$ for charged plate

Things To Know

- Potential Energy : P.E or U or ΔU → P.E
- Electrostatic Potential : V or ΔV or Voltage → E.P
- Work Applied : W_{app}, work done by external agent → W_{appl}
- W_{Field} : Work done by a field → W_{field}

$$\boxed{U_F - U_i = \Delta U = W_{appl} = -W_{Field}}$$

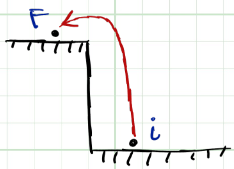
* If work is done by external agent ...

$\Rightarrow W_{appl} > 0 \iff W_{field} < 0$

* If work is done by field ...

$\Rightarrow W_{field} > 0 \iff W_{appl} < 0$

How This Makes Sense

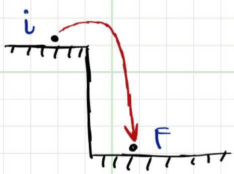


$W_{appl} > 0$

$W_{field} < 0$

$\Delta U > 0$

U increased



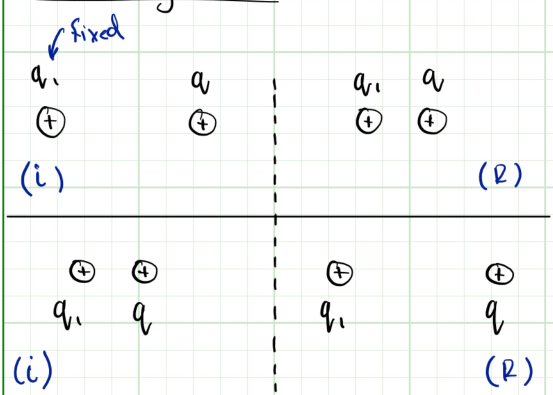
$W_{field} > 0$

$W_{appl} < 0$

$\Delta U < 0$

U decrease

Electromagnetism



$$\left.\begin{array}{l} W_{appl} > 0 \\ W_{field} < 0 \\ \Delta U > 0 \end{array}\right\} \begin{array}{l} U \text{ increased} \\ V \text{ increased} \end{array}$$

$$\Delta V = \frac{\Delta U}{q} = \frac{(+)}{(+)} > 0$$

$$\left.\begin{array}{l} W_{field} > 0 \\ W_{appl} < 0 \\ \Delta U < 0 \end{array}\right\} \begin{array}{l} U \text{ decreased} \\ V \text{ decreased} \end{array}$$

$$\Delta V = \frac{\Delta U}{q} = \frac{(-)}{(+)} < 0$$

$$\left.\begin{matrix} W_{field} > 0 \\ W_{appl} < 0 \\ \Delta U < 0 \end{matrix}\right\} \begin{matrix} U \text{ decreased} \\ V \text{ positive} \end{matrix}$$

$$\Delta V = \frac{\Delta U}{q} = \frac{(-)}{(-)} > 0$$

What is Voltage?

$$\boxed{V_F - V_i = \Delta V = \frac{U_F - U_i}{q} = \frac{\Delta U}{q} = \frac{W_{appl}}{q} = -\frac{W_{field}}{q}}$$

q enters with algebraic sign

$$\Delta V = \left[\frac{J}{C}\right] = [\text{Volt}] = [V]$$

- When a particle moves against the electric field, the change in voltage is positive (voltage increases)

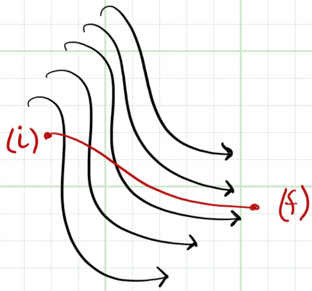
* Charge goes against field = Voltage goes up
* Charge goes along field = Voltage goes down

Equation

$$\Delta V = V_F - V_i = \int_i^F \vec{E} \cdot \vec{ds}$$

Week 2: Lec 1

Electrostatic Potential



$$V_f - V_i = \Delta V = -\int_i^f \vec{E} \cdot \vec{ds}$$

$$\vec{F} = q\vec{E}$$

$$\Delta V = \frac{-W_{field}}{q} \Rightarrow W_{field} = \Delta V(-q)$$

$$W_{field} = \vec{F} \cdot \vec{s}$$

$$dW_{field} = \vec{F} \cdot \vec{ds}$$

$$\int dW_{field} = \int_i^f \vec{F} \cdot \vec{ds}$$

$$W_{field} = \int_i^f \vec{F} \cdot \vec{ds}$$

$$W_{field} = q \int_i^f \vec{E} \cdot \vec{ds}$$

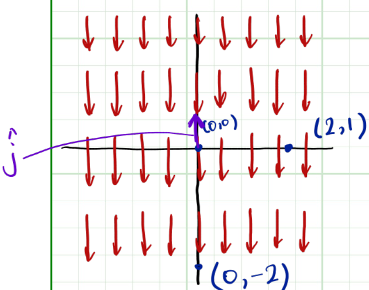
$$\Delta V(-\cancel{q}) = \cancel{q} \int_i^f \vec{E} \cdot \vec{ds}$$

$$\Delta V = -\int_i^f \vec{E} \cdot \vec{ds}$$

$$\boxed{\therefore \Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot \vec{ds}}$$

Example

$$\vec{E} = -|E_0|\hat{j}$$



$$\vec{ds} = d_x\hat{i} + d_y\hat{j} + d_z\hat{k}$$

$$V_{(0,-2)} - V_{(0,0)} = -\int_{(0,0)}^{(0,-2)} \vec{E} \cdot \vec{ds}$$

$$= -\int_{0,0}^{0,-2} |\vec{E}| \cos\angle \vec{E}\cdot\vec{ds}$$

$$= -|E_0| \cos\angle \vec{E}\cdot\vec{ds} \int_{0,0}^{0,-2} dy$$

$$= -|E_0| \overset{-1}{\widetilde{\cos(180)}} \left[y \Big|_0^{-2} \right]$$

$$= -2|E_0|$$

Second Method

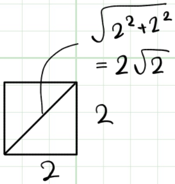
$$V_{0,0} - V_{0,-2} = \Delta V$$

$$= -\int_{0,0}^{0,-2} -|E_0|\hat{j}\cdot dy\hat{j} = |E_0|\int_{y=0}^{y=-2} dy = \boxed{-2|E_0|}$$

Part B = Move to $(0,2)$ and then to $(0,-2)$

(1) $$V_{2,0} - V_{0,0} = -\int_{0,0}^{2,0} |E_0| \cos \cancel{\angle E\cdot ds}^{\,0} \qquad \cos(90^\circ) = 0$$

$$V_{2,0} - V_{0,0} = -\int_{0,0}^{2,0} -|E_0|\hat{j}\cancel{\cdot}^{\,0} dx\hat{i} \qquad \hat{j}\cdot\hat{i} = 0$$



(2) $$V_{0,-2} - V_{2,0} = -\int_{2,0}^{0,-2} -|E|_0 \cos \angle E\cdot ds \; ds$$

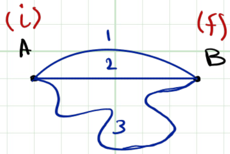
$$= -|E_0| \underbrace{\cos(45^\circ)}_{1/\sqrt{2}} \int_{2,0}^{0,-2} ds$$

$$= -|E_0| \frac{1}{\cancel{\sqrt{2}}} \cdot 2\cancel{\sqrt{2}} = \boxed{-2|E_0|}$$

$$V_{0,-2} - V_{2,0} = -\int_{0,-2}^{20} -|E_0|\hat{j}\cdot[dx\hat{i} + dy\hat{j}] \qquad \hat{j}\cdot\hat{i} = 0 \qquad \hat{j}\cdot\hat{j} = 1$$

$$= |E_0|\int_{y=0}^{y=-2} dy = \boxed{-2|E_0|}$$

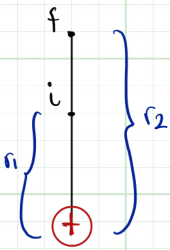
* Electrostatic field is conservative field



It doesn't matter what path you take

Voltage depends only in $\Delta V = V_f - V_i$

Voltage of Point Charge at Distance



$$\Delta V = V_f - V_i = V_{r_2} - V_{r_1}$$

$$= -\int_{r=r_1}^{r=r_2} \vec{E} \cdot \vec{ds}$$

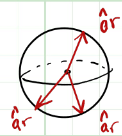
$$= -\int |\vec{E}|\, ds\, \underbrace{\cos \angle \vec{E} \cdot \vec{ds}}_{\cos(0) = 1}$$

$$= -\int_{r=r_1}^{r=r_2} \frac{q}{4\pi\epsilon_0 r^2}\, dr$$

$$= -\frac{q}{4\pi\epsilon_0}\left(-\frac{1}{r}\right)\Bigg|_{r=r_1}^{r=r_2}$$

$$V_{r_2} - V_{r_1} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_2}\right)$$

$$\vec{ds} = ds\,\hat{a}_r = dr\,\underbrace{\hat{a}_r}_{\text{unit vector along radius}}$$



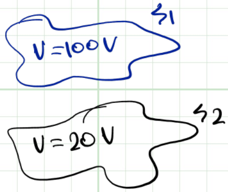
Now let $r_2 \to \infty$, $V_{r_2} = 0$

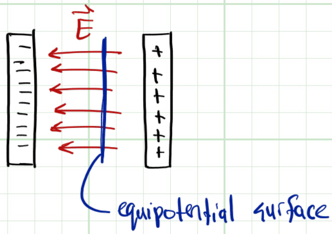
$$-V_{r_1} = -\frac{q}{4\pi\epsilon_0 r} \Rightarrow \boxed{V_q(r) = \frac{q}{4\pi\epsilon_0 r}}$$

Voltage of signed point charge q at a distance r from charge

Equipotential Surfaces

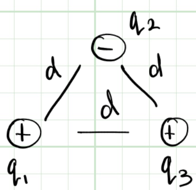
- a surface where voltage is the same anywhere on it.
- voltage doesn't change when moving on field
- Electric field is $\perp$ everywhere to equipotential surface





Electric Potential Energy of System

- The total potential energy of a system of particles is the sum of the potential energies for every pair of particles in the system



$$U = U_{1,2} + U_{1,3} + U_{2,3}$$

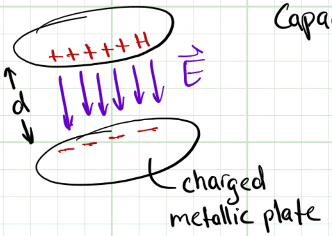
$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{d} + \frac{q_1 q_3}{d} + \frac{q_2 q_3}{d}\right)$$

q enters with algebraic sign

$$\boxed{U = \frac{q_1 q_2}{4\pi\epsilon_0 r}}$$

electric potential energy of 2 charge system

Capacitors and Capacitance



Capacitance $\boxed{C = \frac{Q}{V}}$

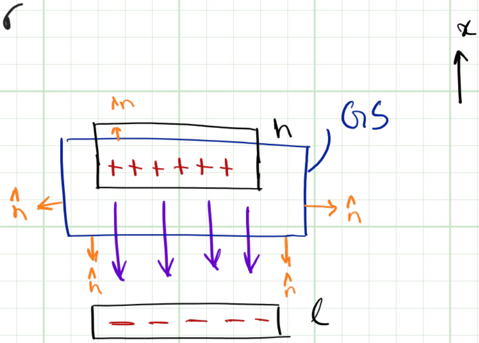
absolute value of charge on one plate

capacitance (always (+))

ΔV : voltage diff btwn 2 plates

* units for capacitance C is $= \frac{[C]}{[V]}$ = Farad = F
* used to store electric energy

Parallel Plate Capacitor



$$\oiint_{GS} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow \underset{\text{bottom surface}}{\iint} E\, dA \cos\angle \hat{E} \cdot \hat{n} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{\varepsilon_0 A}}$$

electric field of capacitor

area of bottom surface

$$EA = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q_{enc}}{\varepsilon_0 A}$$

$$\Delta V = -\int_{x=0}^{x=d} \vec{E}\cdot d\vec{s} = -\int_{x=0}^{x=d} E\,ds\cos\angle\vec{E},d\vec{s} = -\int_{x=0}^{x=d} \frac{Q}{\varepsilon_0 A}\cos(180)\,dx$$

$$= \frac{Q}{\varepsilon_0 A}\int_{x=0}^{x=d} dx = \frac{Qd}{\varepsilon_0 A} \qquad \boxed{\Delta V = \frac{Qd}{\varepsilon_0 A}}$$

$$\Delta V = \frac{Qd}{\varepsilon_0 A} \Rightarrow \left.\frac{\varepsilon_0 A}{d} = \frac{Q}{\Delta V}\right\} \text{capacitance}$$

area of plate analyzed

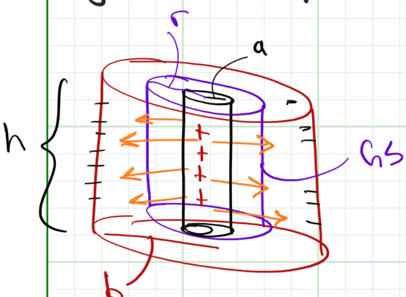
$$\boxed{C_{\parallel} = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}}$$ capacitance of parallel charged plates

$$\boxed{V = Ed}$$

distance bt 2 plates

- electric field unit $E = \frac{[V]}{[m]} = \frac{[N]}{[C]}$

Cylindrical Capacitor



$$\oiint_{\text{side}} \vec{E}\cdot d\vec{A} = \iint_{\text{side}} E\,dA\cos(0) = \frac{Q_{enc}}{\varepsilon_0}$$

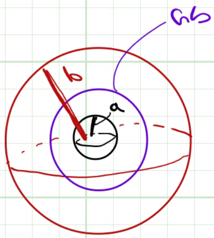
$$\boxed{E = \frac{Q_{enc}}{2\pi r \varepsilon_0 h}}$$ Electric field for cylindrical capacitor radius

$$\boxed{C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 h}{\ln\left(\frac{b}{a}\right)}}$$ capacitance of cylindrical capac. of height h and outer radius b and inner radius a.

$$\boxed{V = \frac{Q}{2\pi\varepsilon_0 h}\ln\left(\frac{b}{a}\right)}$$

$$\boxed{C = \frac{C}{h} = \frac{2\pi\varepsilon_0}{\ln(b/a)}}$$ capacitance of cylindrical capacitor per unit length

Spherical Capacitor



$$\boxed{C = \frac{Q}{V} = 4\pi\epsilon_0 \cdot \frac{ab}{b-a}}$$

Capacitance of spherical capacitor

Isolated Spherical Capacitor

$$\boxed{C = 4\pi\epsilon_0 R}$$

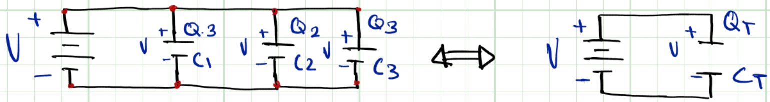
R is radius of spherical conductor

Week 2: Lec 2

Parallel Capacitors

Voltage same across each element

$Q = VC$



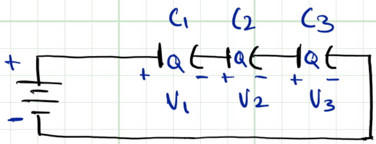
$C_1 \| C_2 \| C_2$

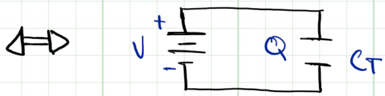
amount of charge is different

$$\boxed{C_T = C_1 + C_2 + C_3}$$

Series Capacitors

Q current is same



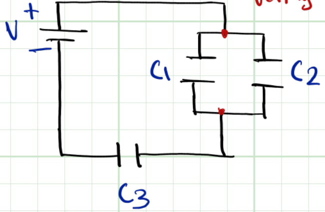


$C_1 \leftrightarrow C_2 \leftrightarrow C_3$

$$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \quad \text{or} \quad \boxed{C_T = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}}$$

Example

same voltage

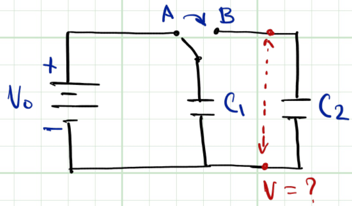


$C_1 \| C_2 = C_1 + C_2$

$(C_1 + C_2) \leftrightarrow C_3$

$$\frac{1}{C_T} = \frac{1}{C_3} + \frac{1}{C_1 + C_2}$$

Example



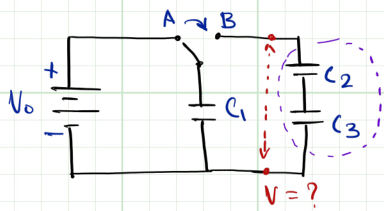
$$Q_0 = C_1 V_0 \quad \text{— config A}$$

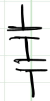
$$Q_0 = C_1 V + C_2 V \quad \text{— config B}$$

$$C_1 V_0 = V(C_1 + C_2)$$

$$V = \frac{C_1 V_0}{C_1 + C_2}$$

the Q must remain same, so to find new voltage, we use Q_0 from config A to find V





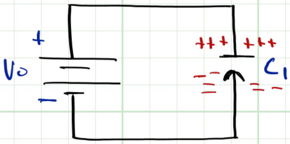
$$C_T = \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_T = \frac{C_2 C_3}{C_2 + C_3}$$

Capacitor Storage

U

$q = CV$

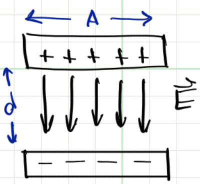


$\Delta U = q\,\Delta V$

$dU = q\,dV$

energy stored in capacitor

$$\boxed{U = \int dU = \int q\,dV = \int C V\,dV = \frac{1}{2} C V^2 = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} qV}$$



- energy is stored in the electric field
- what is energy density?

$$U = \frac{1}{2} C V^2 \Longrightarrow \frac{U}{\text{volume}} = u = \frac{1}{2}\frac{C V^2}{dA}$$

$E^2 = |\vec{E}|^2$

$$= \frac{1}{2}\frac{\epsilon_0 \cancel{A}}{\cancel{d^2}\,\cancel{A}} E^2 \cancel{d^2}$$

$$= \frac{1}{2}\epsilon_0 E^2$$

$$\frac{U}{\text{volume}} = u = \left[\frac{J}{cm^3}\right]$$

$$C = \frac{\epsilon_0 A}{d}$$

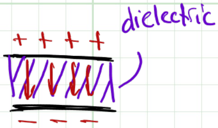
$$V = Ed$$

$$\boxed{u = \frac{1}{2}\epsilon_0 E^2}$$ energy density for parallel plate capacitor

- ϵ_0 is only for vacuum
- we need to modify equation

Dialectric Constant

$$\varepsilon_0 \longrightarrow \underset{\text{dielectric constant}}{\kappa}\,\varepsilon_0 \equiv \overset{\text{relative permitivity}}{\varepsilon_r}\,\overset{\text{permitivity}}{\varepsilon_0}$$



$\kappa = \varepsilon_r$ (relative permitivity)

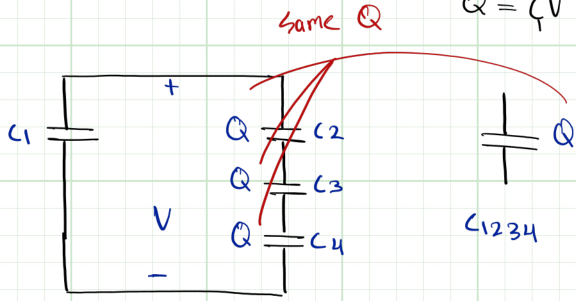
$\varepsilon_r > r$

$$F_{2,1} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{q_1 q_2}{r^2} \qquad C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$E_{q_1} = \frac{q_1}{4\pi\varepsilon_0\varepsilon_r r^2}$$

- What happens when we add a dielectric to capacitor?
  - ↳ C increases because $\kappa = \varepsilon_r > 1$
- When we remove battery, voltage drops and capacitance increases since Q must remain constant : $Q = CV$

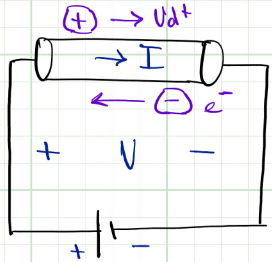
$Q = CV$



Resistance and Resistors

$$\boxed{R = \frac{V}{I}}$$

resistance of a device



- current is the movement of positive charge
- e^- move opp

$$R = \left[\frac{V}{A}\right] = [\Omega]$$

$$\boxed{I = \frac{dq(t)}{dt}}$$

movement of charge is current

$$I = \left[\frac{C}{s}\right] = [A]$$

one of 7 basic units in SI

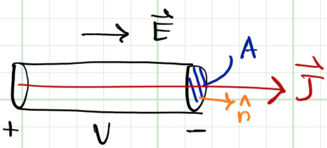
$\vec{J} \triangleq$ current density

by definition

$$\left[\frac{A}{m^2}\right]$$

only place where density referred as m^2

$$I = \iint_A \vec{J} \cdot d\vec{A}$$



- current is the flux of current density through a surface

Assume $\vec{J}$ uniform

$$I = \iint_A \vec{J} \cdot d\vec{A} = JA$$

$$\boxed{I = JA}$$

current is the current density time area

Resistance in Micro Scale

- electrons bump into other things and slow down.
- inhibition of electrons is called resistance

$$R = \frac{V}{I} \iff \rho = \frac{|\vec{E}|}{|\vec{J}|} = \frac{E}{J} \implies \vec{E} = \rho \vec{J}$$

macro scale

resistivity

micro scale

$$\frac{1}{\rho}\vec{E} = \vec{J}$$

$$\boxed{\rho = \frac{E}{J}} = \left[\frac{V\,m}{m\,A}\right] = [\Omega \cdot m]$$

resistivity relates resistance to micro scale

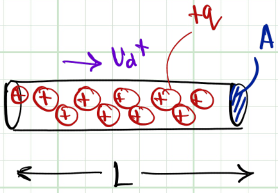
$$\boxed{\sigma \vec{E} = \vec{J}}$$

conductivity

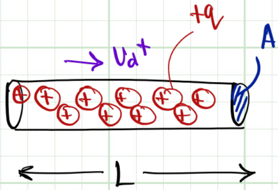
$$\sigma = \left[\frac{1}{\Omega \cdot m}\right]$$

Expression of Current Density

- drift velocity



- current is movement of charge



N : density

$$\boxed{Q = NLAq^+}$$ ← total charges

We know $L = V_d t$

$$\boxed{Q = NV_d t A q^+}$$ total charges in terms of V_d

$$\frac{Q}{t} = NV_d A q^+$$

We know $\frac{Q}{t} = I$

$$I = NV_d A q^+$$

$$\boxed{\frac{I}{A} = J^+ = NV_d q^+}$$ current density related to drift density

$$\vec{J}^+ = N\vec{V}_d q^+$$

$$\boxed{\vec{J}_T = \vec{J}^+ + \vec{J}^- = N^+\vec{V}_d^+ q^+ + N^-\vec{V}_d^- q^-}$$

$N^{+\text{ or }-}$ density for (+)ve or (-)ve charges

q^+ V_d^+

⊕ → ← $\vec{J}^+ = N^+ q^+ \vec{V}_d^+$ (N^+) and $q^+ > 0$

V_d^-

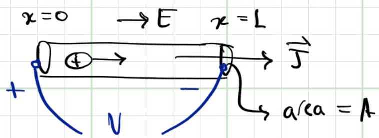
q^- ← ⊖ → $\vec{J}^- = N^- q^- \vec{V}_d^-$ $(N^-) > 0$ but $q^- < 0$

$\therefore \vec{J}_T = \vec{J}^+ + \vec{J}^- \longrightarrow$ same direction

Finding Resistance of Wire

$$\boxed{R = \rho \frac{L}{A}}$$

resistance of a wire length L, cross-sectional area A made of resistivity ρ



$$R = \frac{V}{I} \qquad V = EL \qquad I = JA$$

$$R = \frac{V}{I} = \frac{EL}{JA} = \rho \frac{L}{A}$$

and since $\rho = \frac{E}{J}$

Power Dissipated in Resistor

$$\Delta V = \frac{\Delta U}{q} \Longrightarrow q\,dV = dU$$

we know $V = IR$

$$\frac{dV}{dI} = R \longrightarrow dV = R\,dI$$

$$\int q\,dV = \int dU$$

$$\int q\,R\,dI = \int dU$$

we know rate change of energy is given by power

$$P = \frac{\Delta U}{\Delta t}$$

$$qRI = U$$

$$U = qRI$$

$$\frac{dU}{dt} = \frac{dq}{dt} RI = IRI$$

$$\boxed{P = I^2 R}$$

P is dissapated power of a resistor

$$\boxed{P = I^2 R = \frac{V^2}{R} = IV}$$

any device ... relating V, I, and power

power: [Watt] = [W] = [A·V] ampere - Volt

Resistors in Parallel

$I = I_1 + I_2 + I_3$ ← KCL

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Resistors in Series

KUL

- same current
- different voltage $V = V_1 + V_2 + V_3$

$$\boxed{R_T = R_1 + R_2 + R_3}$$

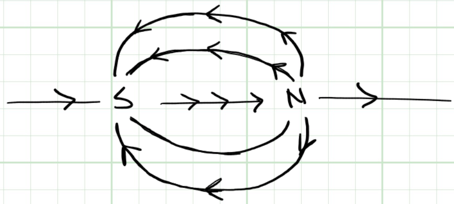
Midterm Review

Magnetic Field ($\vec{B}$)

$\vec{E} = \dfrac{\vec{F}}{q}$ ← electric force

$\vec{B}$

$\vec{F}_B$ ← magnetic force



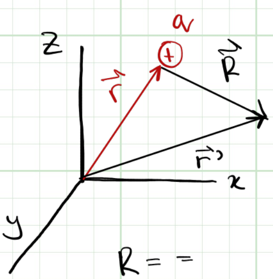
- magnetic field always starts at North pole and ends at South pole

Magnetic Flux : $\boxed{\oiint \vec{B} \cdot d\vec{A} = 0}$ Gauss Law for magnetic flux

reason why magnetic field lines always loop and close on themselves

- Movement of charge (current) causes magnetic fields.
- Fundamentally, related to magnetic moment (fundamental property of things... like mass, charge etc.)

Biot-Savart Law

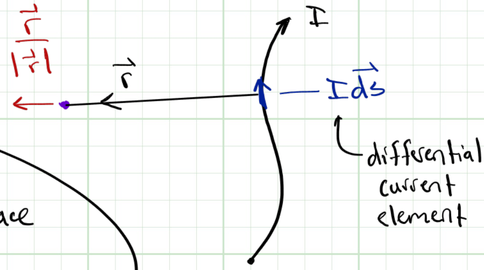


$\hat{a}_R = \dfrac{\vec{R}}{|\vec{R}|}$

$\vec{E} = \dfrac{q}{4\pi\epsilon_0 R^2}\hat{a}_R$

$R = =$

$$\vec{dB} = \frac{\mu_0 I ds \times \hat{a}_r}{4\pi r^2} \qquad \hat{a}_r = \frac{\vec{r}}{|\vec{r}|}$$



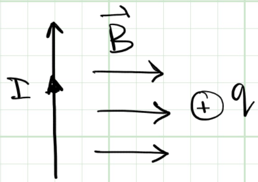
$\mu_0 \triangleq$ permeability of free space

$\mu_0 = 4\pi \times 10^{-7}$

Integrate

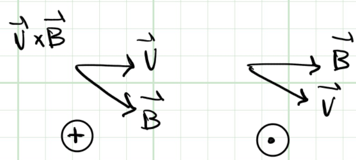
$$B = \int \vec{dB} = \int \frac{\mu_0 I\vec{ds} \times \hat{a}_r}{4\pi r^2}$$ Biot-Savart Law

Units



- $\vec{F}_B = 0$ if q doesn't move
- $\vec{F}_B = 0$ if q moves paralel to $\vec{B}$

$$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$$ magnetic force



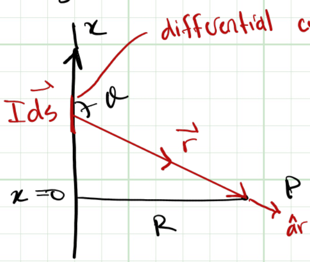
$$\vec{B} = \left[\frac{N \cdot s}{C \cdot m}\right] = \text{Tesla} = [T]$$ unit for magnetic field $\vec{B}$

$$= \frac{\cancel{C} \cdot V \cdot s}{m \cdot \cancel{C} \cdot m} = \left[\frac{V \cdot s}{m^2}\right] = \left[\frac{\text{Weber}}{m^2}\right]$$

$$\mu_0 = \frac{T \cdot m^{\cancel{2}}}{A \cdot \cancel{m}} = \left[\frac{T \cdot m}{A}\right] = \frac{V \cdot s \cdot \cancel{m}}{m^{\cancel{2}} \cdot A} = \left[\underbrace{\frac{V \cdot s}{A}}_{\text{Henry}} \cdot \frac{1}{m}\right]$$

$\mu_0 = \left[\frac{\text{Henry}}{m}\right] = \left[\frac{H}{m}\right]$ unit for μ_0 ~ permeability of vacuum

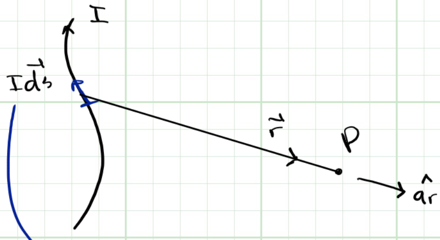
Magnetic Field at distance r from Current



$$\vec{B} = \int_{x=-\infty}^{x=\infty} \frac{\mu_0 I\vec{ds} \times \hat{a}_r}{4\pi r^2}$$

Week 4, Lec 1

- Biot-Savart Law

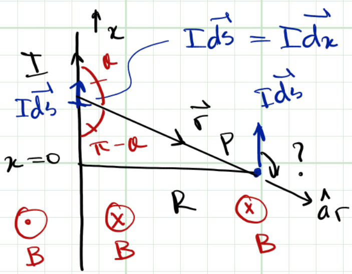


differential current element

$$\vec{B} = \int \frac{\mu_0}{4\pi r^2} I\vec{ds} \times \hat{a}_r$$

$$\hat{a}_r = \frac{\vec{r}}{|\vec{r}|}$$

$$I\vec{ds} = I\vec{dx} = I\,dx\,\hat{i}$$



$$\vec{B} = \int_{x=-\infty}^{x=+\infty} \frac{\mu_0 I\vec{dx} \times \hat{a}_r}{4\pi r^2}$$

$$B = \frac{\mu_0}{4\pi} I \int_{-\infty}^{\infty} \frac{dx}{r} \sin\theta$$

$$\sin\theta = \sin(\pi-\theta) = \frac{R}{r}$$

$$r = \sqrt{x^2 + R^2}$$

$$r^3 = (x^2+R^2)^{3/2}$$

$$B = \text{"} \int_{-\infty}^{\infty} \frac{dx}{r} \cdot \frac{R}{r}$$

$$= \text{"} \int_{-\infty}^{\infty} \frac{dx\,R}{r^3}$$

constant

$$= \text{"} \int_{-\infty}^{\infty} \frac{dx\,R}{(x^2+R^2)^{3/2}}$$

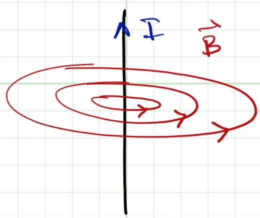
even, so

$$\int_{-\infty}^{\infty} x^2 = 2\int_0^{\infty} x^2$$

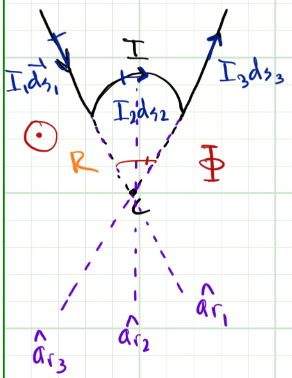
$$B = \frac{\mu_0 I}{2\pi R} \underbrace{\left[\frac{x}{\sqrt{x^2+R^2}}\right]_0^{\infty}}_{=1-0}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R}}$$

magnetic field at distance R



Less Symmetry Example



What is $|\vec{B}|$ at centre c?

$$I_1 = I_2 = I_3$$

$$\vec{dB_1} \propto I_1 \vec{ds_1} \times \hat{a}_{r_1} = 0 \qquad \sin(0) = 0$$

$$\vec{dB_3} \propto I_3 \vec{ds_3} \times \vec{a}_{r_3} = 0 \qquad \sin(180) = 0$$

$$\vec{dB_2} \propto I_2 \vec{ds_2} \times \vec{a}_{r_2} = 1 \qquad \sin(90) = 1$$

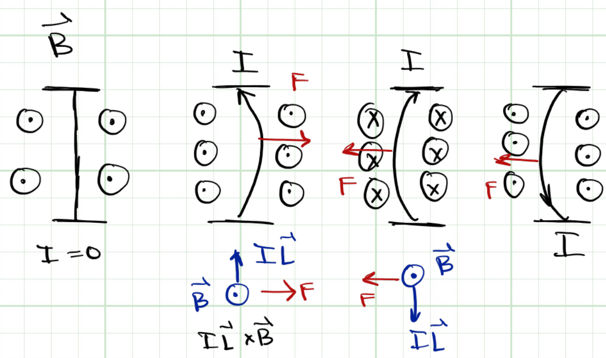
$$\vec{B}_2 = \int \frac{\mu_0 I_2 \vec{ds_2} \times \hat{a}_{r_2}}{4\pi R^2}$$

$$\vec{B}_2 = \int_{\Phi} \frac{\mu_0 I_2 ds_2}{4\pi R^2} = \int_{\phi=0}^{\phi=\Phi} \frac{\mu_0 I R d\phi}{4\pi R^2} = \frac{\mu_0 I}{4\pi R} \phi \Big|_0^{\Phi}$$

$$\boxed{\vec{B}_2 = \frac{\mu_0 I \Phi}{4\pi R}}$$

magnitude of $\vec{B}$ at distance R and angle Φ in radian

Magnetic Force Experiment



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$d\vec{F} = dq\, \vec{v} \times \vec{B}$$

$$d\vec{F} = I\cancel{dt}\, \frac{d\vec{L}}{\cancel{dt}} \times \vec{B}$$

$$I = \frac{dq}{dt} \Longrightarrow dq = I\,dt$$

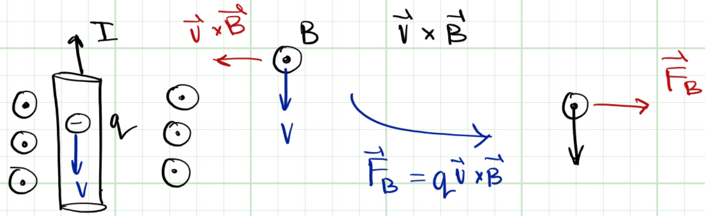
$$\vec{v} = \frac{dL}{dt}$$

assume wire straight, and $\vec{I}$ not depending on space

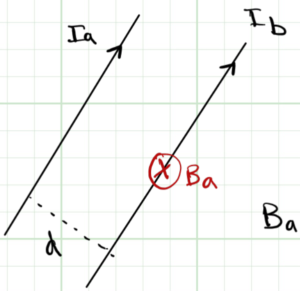
$$\vec{F} = \int I\, d\vec{L} \times \vec{B} \Longrightarrow \boxed{\vec{F}_B = I\vec{L} \times \vec{B}}$$

cross product to find $\vec{F}_B$

Microscopic Scale



the negative charge q switches direction of $\vec{v} \times \vec{B}$



same direction I, attracts

opposite direction I, repells

$$B_a = \frac{\mu_0 I_a}{2\pi d} \qquad F_{ba} = I_b \vec{L} \times \vec{B}_a$$

Week 4: Lecture 2

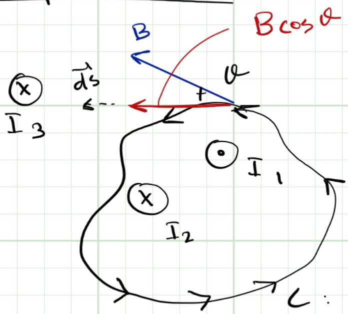
$$\boxed{|F_{b,a}| = \frac{\mu_0 L \, I_L I_b}{2\pi d}}$$

Force on wire when it and another wire have current a, b and distance d bt. them

- Find the $\vec{B}$ first, the the $\vec{F}$

- after finding the $\vec{B}$ at the wire, use RHR crossing current of the wire you're at (not the other ones) to find the direction of the $\vec{F}_B$ from each $\vec{B}$ on the wire you're at.

Ampere's Law



C : Ampeian loop

$$\boxed{\oint_C \vec{B} \cdot \vec{ds} = \mu_0 I_{enc}}$$

closed current has algebraic sign

line integral of $\vec{B}$ over the closed path C (also called circulation of B) is equal to μ_0 times the current enclosed by C

$$\oint_S \vec{B} \cdot \vec{ds}$$

differential length always tangential

$$\vec{B} \cdot \vec{ds} = \underbrace{B\cos\theta}_{\text{tangential component of B}} \, ds$$

" tangential component times ds gives the enclosed current (multiplipied by μ_0) "

Sign of I

Book: fingers along C, thumb is (+)ve current

Prof: Put thumb along current, fingers go along path C

↳ if fingers opp. C, I (-)ve

↳ " " same C, I (+)ve

∴ in eg. above

I_1 (+)ve

I_2 (-)ve

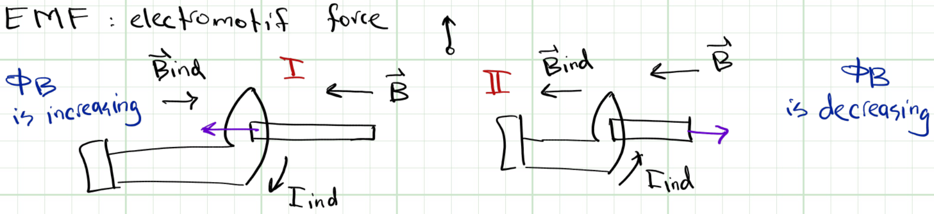
Solenoids

- thumb along direction of current, fingers show $\vec{B}$ inside coil.
- no $\vec{B}$ outside solenoid

$\boxed{B = \mu_0 n I}$ magnetic field inside solenoid, where n is # of wire turn/loops per unit length

Faraday Law, Lenz Law

- You'll have current only if there is a Δ magnetic flux
- Induced Current (I_{ind}) : current caused by magnetic field
- EMF : electromotif force



- I_{ind} causes $\vec{B}_{ind}$
- $\vec{B}_{ind}$ sometimes aids and sometimes hinders $\vec{B}_{magnet}$

$$\boxed{\mathcal{E} = EMF = -N\underbrace{\frac{d}{dt}\Phi_B}_{} = -N\frac{d}{dt}\iint \vec{B}\cdot d\vec{A}}$$

$-$: I_{ind} and EMF opposes $\Delta\Phi_B$

N: number of loops in coil

$\frac{d}{dt}\Phi_B$: time derivative of magnetic flux

- I_{ind} opposes the change in magnetic flux
  - ↳ I : Φ_B is increasing, I_{ind} is (-)ve
  - ↳ II : Φ_B is decreasing, I_{ind} is (+)ve

EMF (9.3)

- $EMF = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{A}$
- Unit for EMF is volts (it's a kind of voltage)
- EMF created even if the wire doesn't exist, since EMF only depends on change in magnetic flux.
- EMF is "induced electric field"

$$\mathcal{E} = \oint_c \vec{E} \cdot d\vec{s} \qquad \text{v.s.} \qquad \Delta V = -\int_i^R \vec{E} \cdot d\vec{s}$$

over a closed loop

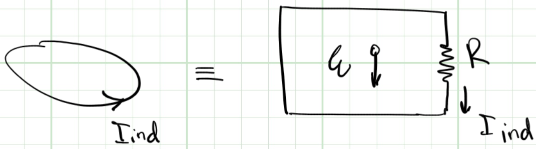
- EMF is not the same as a regular voltage (in electrostatics)
- You can change flux by changing angle b.t. $\vec{B}$ and $d\vec{A}$

For Electrostatic Voltage

$$\Delta V = -\int \vec{E} \cdot d\vec{s} \longrightarrow \oint \vec{E} \cdot d\vec{s} = 0$$

But for EMF

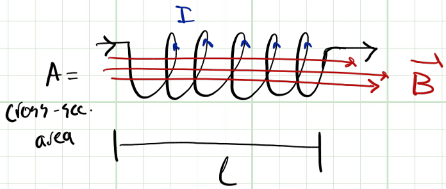
$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \Phi_B = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{A} \rightarrow \text{can't be zero}$$



Week 5: lec 1

Inductance

- inductance of inductor
- self induction
- energy stored in inductor
- Voltage across an inductor
- ∥ and ⟷ connection
- solenoid ≡ inductor
- stores magnetic energy

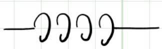


$N \triangleq$ total # of loops

$n \triangleq$ number of loops per unit length $[1/cm]$

$\therefore n = \frac{N}{l} \Rightarrow N = nl$

Inductance

 $B = \mu_0 n I \Rightarrow \Phi_B = BA = \mu_0 n I A$

inductance definition

$N\Phi_B$ called magnetic flux linkage

$$\boxed{L = \frac{N\Phi_B}{I}} = \frac{N \mu_0 n \cancel{I} A}{\cancel{I}} = nl\mu_0 nA = n^2 A l \mu_0$$

$$\boxed{L = n^2 A l \mu_0}$$

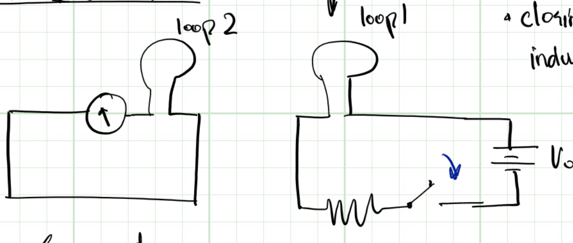
$$L = \left[\frac{1}{m} \cdot m^2 \cdot \frac{H}{m}\right] = [H]$$

henry

inductance for inductor length l, n is # loops/unit length and cross-sec. Area A.

Self Induction

self induced EMF $= \mathcal{E}_{ind} = \mathcal{E}_L$



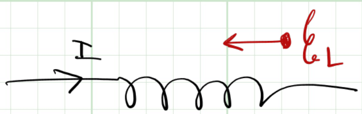
- closing and opening switch induces EMF on loop 2

$$\mathcal{E} = -\frac{d}{dt} N\Phi_B$$

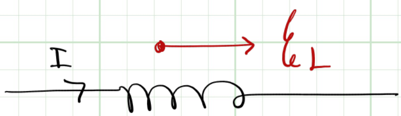
- When you close the switch, current starts flowing through loop 1.
  ↳ $\Delta\Phi_B$ penetrates loop 2, causing ε EMF
- The I_{ind} in Loop 2 causes another I_{ind} / EMF in loop 1
  ↳ call this self induced EMF

$$\text{self induction} = \varepsilon_{\substack{\text{self}\\ \text{induced}}} = \varepsilon_L = -\frac{d}{dL} N\Phi_B$$

- ε_L opposes the change in loop 1 itself.

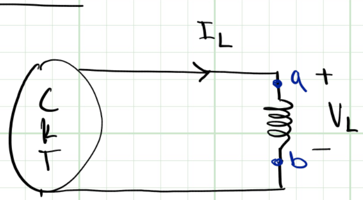


I is increasing
B is increasing
Φ_B is increasing
ε_L opposes current



I is decreasing
B is decreasing
Φ_B is decreasing
ε_L aids current

<u>Circuits</u>



- can't do Voltage <u>inside</u> inductor

Find expression for V_L

$$V_L = L\frac{dI_L}{dt} \qquad [H] = \left[\frac{V\cdot s}{A}\right]$$

$L = \frac{N\Phi_B}{I}$, also $\mathcal{E}_L = -\frac{d}{dt} N\Phi_B$

• inductance depends on geometry

$N\Phi_B = LI$

so $\mathcal{E}_L = -\frac{d}{dt} LI = -L\frac{dI}{dt}$

assuming time doesn't affect

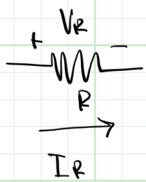
inductance

$$\boxed{\mathcal{E}_L = -L\frac{dI_L}{dt}}$$

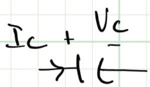
$$\boxed{V_L = -\mathcal{E}_L = L\frac{dI}{dt}}$$

Voltage across inductor with inductance L

- Inductor is differentiator for current
- Integrate V_L to get charge



$$\boxed{V_R = I_R I}$$

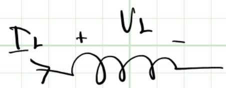


$Q = CV_c$

$\frac{dQ}{dt} = C\frac{dV_c}{dt}$

$$\boxed{I_c = C\frac{dV_c}{dt}}$$

(open to DC
no current



$$\boxed{V_L = L\frac{dI_L}{dt}}$$

("short" to DC
no voltage

Inductor Energy Storage

$I = \frac{dq}{dt}$ $\quad dq = I\,dt$

$dU = dq\,V$

$dU = I\,dt\,V$

$dU = I\cancel{dt}\,L\frac{dI}{\cancel{dt}}$

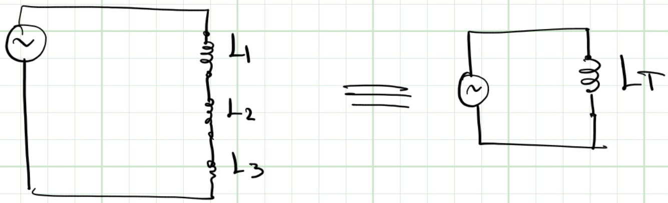
$\int dU = \int IL\,dI$

$$\boxed{U = \frac{1}{2}LI^2}$$

energy stored in inductor with current I and inductance L

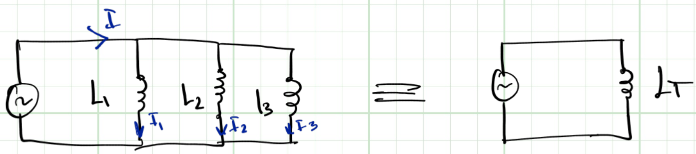
Series current same

$V = V_1 + V_2 + V_3$ (KVL)



$$\boxed{L_T = L_1 + L_2 + L_3}$$

Parallel voltage same



$I = I_1 + I_2 + I_3$ (KCL)

$$L_T = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)^{-1}$$

Week 5 : lec # 2

$V_L = L \frac{dI}{dt}$ ← if $\frac{dI}{dt} = 0$, $V_L = 0$, so short

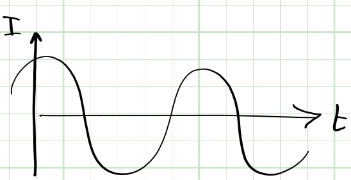
$I_c = C \frac{dV_c}{dt}$ ← if $\frac{dV_c}{dt} = 0$, $I_L = 0$, so open

$V_R = R I_R$

Applications

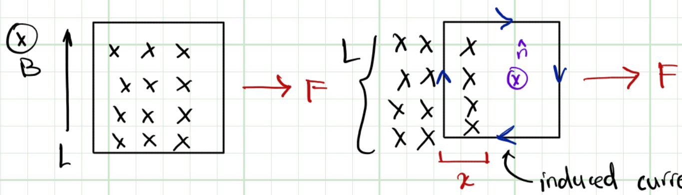
$$\mathcal{E}_L = -N \frac{d}{dt} \Phi_B = -N \frac{d}{dt} \iint_S \vec{B} \cdot d\vec{A}$$

- either increase area or
- change angle → generator loop rotating
- in generator, angle bt coil and $\vec{B}$ changes sinusoidally, so does voltage as a result



Mehanical → Electrical

assume V is constant (no a)



induced current because magnetic flux is changing

- area is changing with time
- the induced $\vec{B}$ must aid the original $\vec{B}$, so the $\vec{B}_{ind}$ should go into page and $\therefore$ I_{ind} is clockwise

velocity

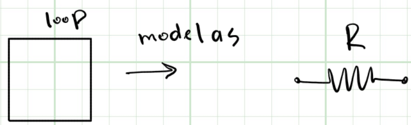
$$\boxed{|\mathcal{E}_0| = \left|-\frac{d}{dt}(BxL)\right| = \left|BL\frac{dx}{dt}\right| = BLV}$$

velocity

since $\mathcal{E} = IR$

$$\boxed{|I_{ind}| = \frac{|\mathcal{E}|}{R} = \frac{BLV}{R}}$$

I_{ind} with B and L and velocity, R resistance



recall $P = I^2R$

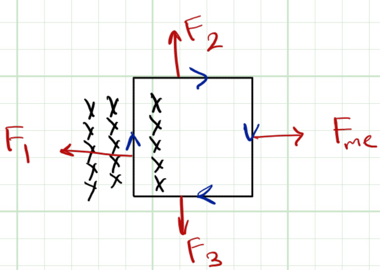
$P_{diss} = I_{ind}^2 R$

$$\boxed{P_{diss} = \frac{B^2L^2v^2}{R}}$$

power dissopation with B, L, and vlocity V

$$\boxed{\frac{dW}{dt} = P = \vec{F}\cdot\frac{d\vec{x}}{dt} = \vec{F}\cdot\vec{v}}$$

work related to power, force and velocity.



since we said no $\vec{a}$,

$F_{net} = 0$

$F_1 + F_2 + F_3 + F_{me} = 0$

- F_2 and F_3 cancel each other out up $\leftrightarrow$ down.
- so $|F_{me}| = |F_1|$
- I know what $|F_1|$ is since it's a current-carrying wire going through $\vec{B}$, so $|F_1| = I \vec{L} \times \vec{B}$, and we know angle $= 90°$

I_{ind} $\qquad$ $\sin(90) = 1$

$$F_1 = I_{ind} \vec{L} \times \vec{B}$$

$$\therefore F_1 = F_{me} = \frac{B^2 L^2 v}{R}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos(0) = Fv$$

Week 5 : Lecture 3

Eddy Currents

- magnetic fields induced when change in magnetic flux induces current called eddy currents (messy whirlpools).

Figure shows a rectangular wire loop of total resistance R of dimensions H and L placed inside a non-uniform time varying magnetic field given by $\vec{B} = A_0 x \sin(\omega t)\ \hat{k}$, where A_0 and ω are positive constants.

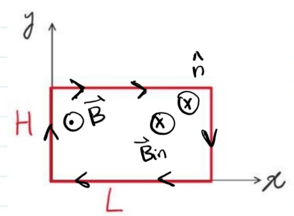
a) What is the direction of EMF (clockwise or counterclockwise) induces in the wire for $0 < \omega t < \pi/2$. Justify your answer. **[2Pts]**

b) What is the magnitude of the EMF as a function of H and L (and other parameters) for all times? **[4Pts]**

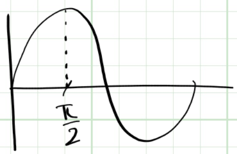
c) What is the magnitude of the induced current as a function of H and L (and other parameters) for all times? **[2Pts]**

d) How much power is dissipated in the wire? **[2Pts]**

e) Who or what is proving the dissipated power? **{Shall I ask this question ?}**



a)



For $0 < \omega t < \frac{\pi}{2}$, sin function is increasing, meaning Φ_B increases. To oppose the increasing Φ_B, $\vec{B}_{ind}$ must be into page and

$\boxed{\therefore \text{ direction of EMF is CW}}$

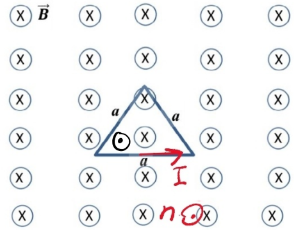
b)
$$\Phi_B = \iint A_0 x \sin\omega t\, dx\, dy \cos(180)$$
$$= -A\sin\omega t \int_0^L x\, dx \int_0^H dy$$
$$= -A\sin\omega t \left(\frac{x^2}{2}\right)_0^L (y)_0^H$$
$$= -A\sin\omega t \frac{L^2}{2} H$$

$$|\mathcal{E}| = \left|-\frac{d}{dt}\Phi_B\right| = \left|\frac{d}{dt}\left(A_0 \sin\omega t \frac{L^2}{2} H\right)\right| = \frac{A_0 L^2 H\omega}{2} |\cos\omega t|$$

c) $$|I_{ind}| = \frac{|\mathcal{E}|}{R} = \frac{A_0 L^2 H\omega |\cos\omega t|}{2R}$$

d) $P_{diss} =$

Figure shows a metallic wire loop in the shape of an equilateral triangle (all sides are equal having the length a centimeter) immersed in a magnetic field $\vec{B}$. The magnetic field ($\vec{B}$) is uniform in space, pointing into the page, but changes with time according to $t\,e^{-t}$ (T). Considering the time interval $0 < t < 1$ second, answer the following questions.

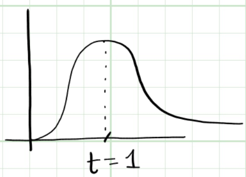


a) Derive an expression for the magnitude of the induced current in terms of a and t, assuming each side of the triangle has a resistance R (Ω). What is the direction of the induced current (clockwise or counter clockwise)? [6 Pts]

b) Obtain an expression for the total power dissipated in the metallic loop. Show all your work. (Try to simplify your answer as much as possible). [2Pts]

c) Suppose that each side of the triangle is doubled in length, while the loop is still entirely immersed in the magnetic field. As compared to the part (a), does the induced current increase or decrease and by how much? Show all your work.]2Pts]

a) $B(t) = te^{-t}$



For $0<t<1$, $\vec{B}$ is increasing, so I_{ind} must be CCW so that B_{ind} opposes $\Delta \Phi_B$

$$|\mathcal{E}| = \left| -\frac{d}{dt}\Phi_B \right|$$

$$\Phi_B = \iint te^{-t}\, dA \cos 180 = -te^{-t} A \qquad A = \frac{\sqrt{3}a^2}{4}$$

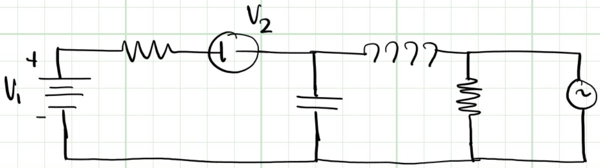
$$\Phi_B = -te^{-t}\frac{\sqrt{3}a^2}{4}$$

$$|\mathcal{E}| = \left| -\frac{d}{dt}\Phi_B \right| = \frac{d}{dt}\left(-te^{-t}\frac{\sqrt{3}a^2}{4}\right) = \frac{a^2}{4\sqrt{3}R}\left(1-te^{-t}\right)$$

$$I_{ind} = \frac{|\mathcal{E}|}{R} =$$

Week 6: lec 1

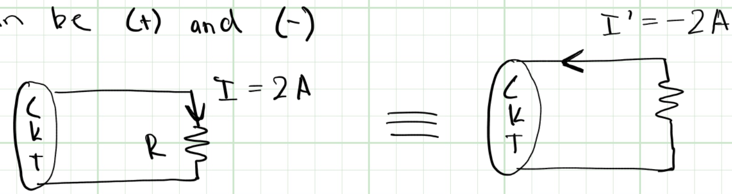
- Solving ckt → find V and I for every element



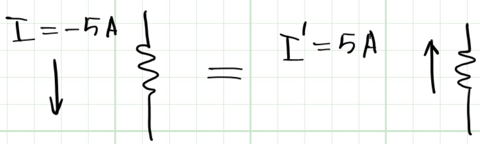
- ckt: collection of electrical elements, lets charges move
  - provides path for current
  - does info processing

Current in Ckt

- can be (+) and (-)

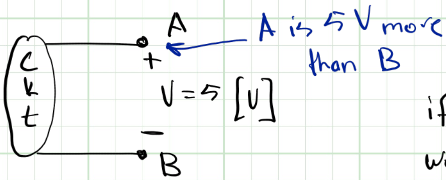


- current shows movement of (+)ve charges



switch side
switch sign

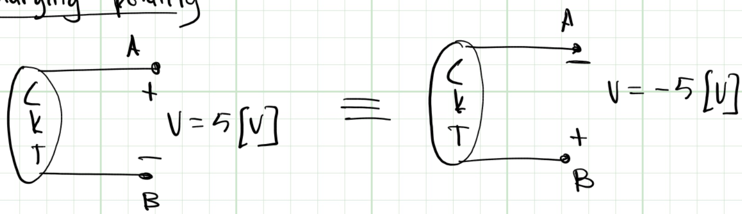
Voltage in Ckt



if	$A = 5$	$A = 0$	$A = -5$
	$B = 0$	$B = -5$	$A = -10$

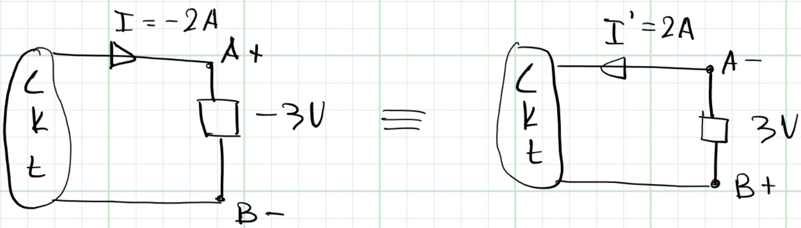
if 1 C moves $A \to B$,
we will lose 5J of energy

Charging polarity



swith polarity, switch sign

Equivalent CKts

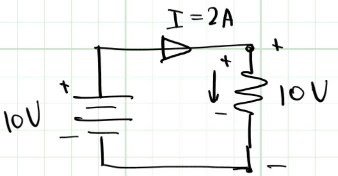


Convention for Power

$P = IV$ "dissapated power is (+)"

if element dissipates powe, power is (+)

if element supplies power, power is (−)



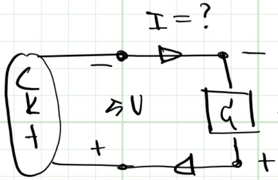
dissipated power > 0

supplied power < 0

* if positive current enters (+) terminal of element, that power must be dissapated, or (+)

* if positive current leaves the (+) terminal, that power is supplied, or (−)

Eg:



$P_q = +40$ [W]

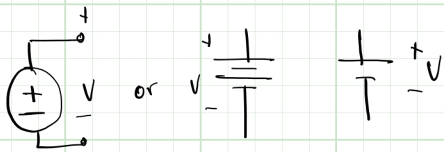
$$|I| = \frac{|P|}{|V|} = \frac{40}{5} = 8$$

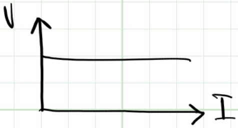
since (+) current leaves (+) element, so power is supplied, so it's negative

$\therefore I = -8$ A

Voltage Source

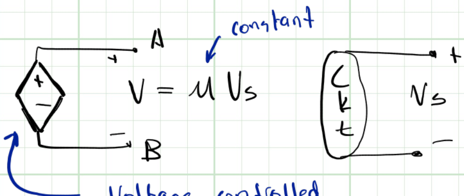
* independent Voltage source:





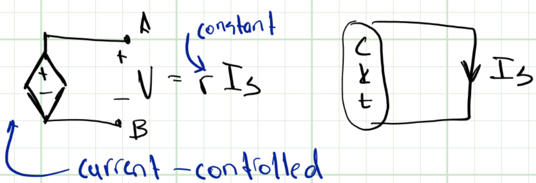
ideal " gives us same voltage no matter how many elements we add or how much current we draw from it

* dependent Voltage source:



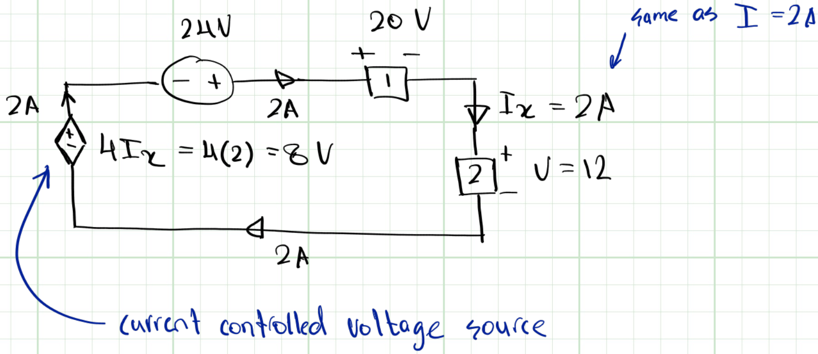
Voltage controlled Voltage soure

voltage can eighter depend on the current somewhere else or voltage elsewhere



current-controlled Voltage source

Eg:



Power in (− +) 24V

$|P| = IV = 2 \times 24 = 48\ W$

since (+) current leaves (+) terminal, power is (−) and supplied
or enters (−)

$P_{24} = -48\ [W]$

Power in 1 (+ −) 20V

$|P| = IV = 2 \times 20 = 40$

(+) I enters (+) terminal, so power is (+) and ∴ dissapated

$\therefore P_1 = +40\ [W]$

Power in 2 (+ −) V=12

$|P| = IV = 2 \times 12$ $\qquad P_2 = 24\ [W]$

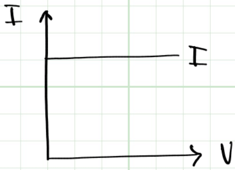
Power in (+ −) V=8V

$|P| = 2 \times 8 = 16$ $\qquad P = -16\ [W]$

Week 6: Lec 2

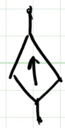
Current Sources

Independent Current Source



Provides constant

Dependent Current Source

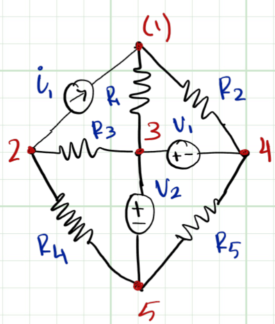


↳ Voltage-controlled current source $\quad I = g\,V_s$

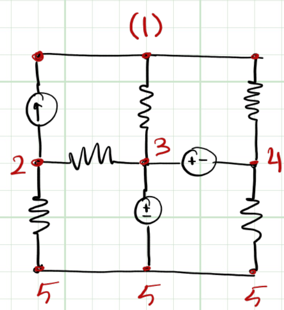
↳ current-controlled current source $\quad I = \beta\, i_s$

Nodes, Branches, Nodes

* **node**: a point in a circuit where 2 or elements come together



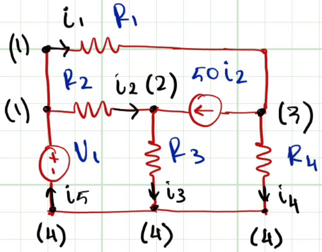
$\equiv$



* **Branch**: a part of a circuit that contains an element.
* **Loop**: a closed path, you start somewhere and end up the same place without visiting a node more than once

Kirchoff Current Law

* algebraic sum of the current of each node is 0.
* for each node in ckt, current entering = current leaving



* enter i values w/h algebraic sign
* 4 equations, but one of them is extra

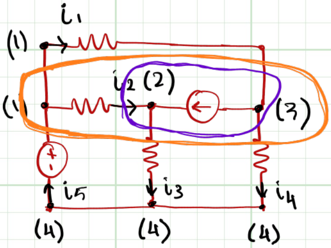
Node 1: $i_5 = i_1 + i_2$

Node 2: $i_2 + 50i_2 = i_3$

Node 3: $i_1 = 50i_3 + i_4$

Node 4: $i_3 + i_4 = i_5$

Applying KCL to a closed surface



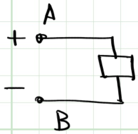
Surface Purple : $i_2 + i_1 = i_3 + i_4$

Orange : $i_5 + \cancel{i_1} = i_3 + i_4 + \cancel{i_1}$

$i_5 = i_3 + i_4$

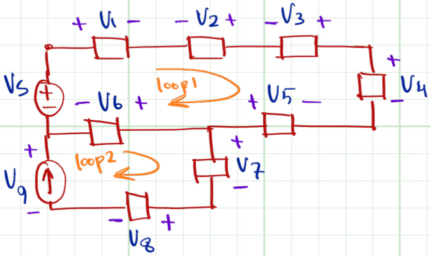
Kirchhoff Voltage Law

* algebraic sum of voltages along loop is zero.
* Potential Drop / Decrease : A → B
* Potential Rise / Increase : B → A



* As u go around loop,

potential increase = potential decrease



Loop 1

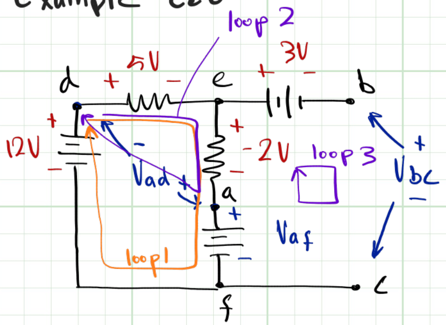
$$V_2 + V_3 + V_5 + V_5 = V_1 + V_4 + V_6$$

Loop 2

$$V_6 + V_9 = V_7 + V_8$$

* enter values where they are with their algebraic sign.

Example Ckt



Loop 1

$PI = PD$

$12 = 5 - 2 - V_{af}$

$V_{af} = 9\,V$

Loop 2

$PI = PD$

$0 = 5 - 2 + V_{ad}$

$V_{ad} = -3\,V$

Loop 4

$PI = PD$

$V_{af} - 2 = 3 + V_{bc}$

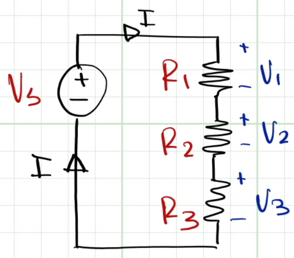
$V_{bc} = V_{af} - 2 - 3$

$= 9 - 2 - 3$

$V_{bc} = 4\,V$

Note that point c and f are the same node since they have no elements between them.

Voltage Divider Ckt



$PI = PD$

$V_s = V_1 + V_2 + V_3$

$V_s = IR_1 + IR_2 + IR_3$

$V_s = I(R_1 + R_2 + R_3)$

$$I = \frac{V_s}{R_1 + R_2 + R_3}$$

got current, can find V_1, V_2, V_3

$$\therefore V_1 = IR_1 = \frac{V_s R_1}{R_1 + R_2 + R_3}$$

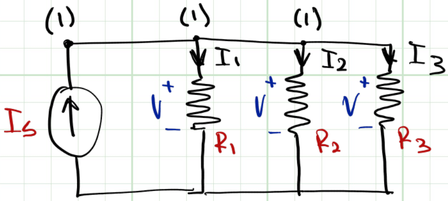
$$V_2 = IR_2 = \frac{V_s R_2}{R_1 + R_2 + R_3}$$

$$V_3 = IR_3 = \frac{V_s R_3}{R_1 + R_2 + R_3}$$

$$\boxed{V_k = V_s \cdot \frac{R_k}{\sum_{j=1}^{N} R_j}}$$

source voltage

Current Divider Ckt



$$CE = CL$$

$$I_s = I_1 + I_2 + I_3$$

$$I_s = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$V = I_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$I_1 = I_s \frac{1/R_1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

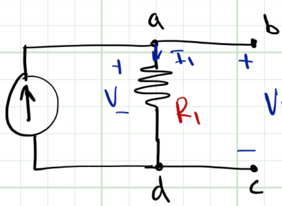
$$\boxed{I_k = I_s \frac{\frac{1}{R_k}}{\sum_{j=1}^{N} \frac{1}{R_j}}}$$

$$I_2 = I_s \frac{1/R_2}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$I_3 = I_s \frac{1/R_3}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Open Ckt Condition

- something with infinite resistance letting $R_2 \to \infty$
- has voltage across it, but no current flows through

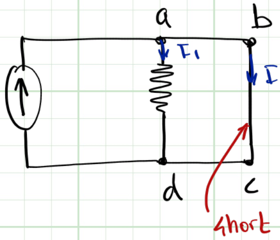


$$V_{bc} = V_{open}$$

$$\boxed{V_{bc} = V_{open} = V_{ad} = I_1 R_1 = I_s R_1 = V}$$

Short Ckt Condition

- has zero resistance
- there is a current through a short, but no voltage across it.

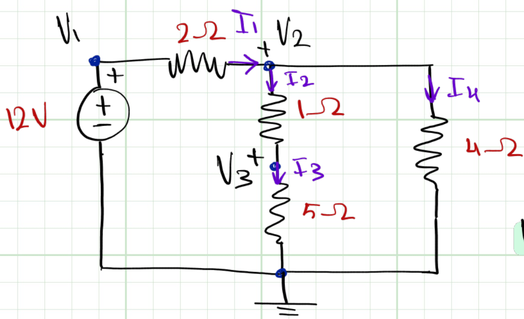


$I_1 = 0$, $I_2 = I_s$

$$\boxed{V_{ab} = V_{bc} = V_{short} = I_1 R_1 = 0}$$

Nodal Analysis

- Variable we solve is Voltage at nodes
- The law that you apply is KCL



Let N be # of nodes
N_v be # of voltage sources

# of eqn needed

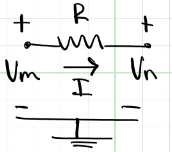
$\boxed{N-1-N_v} = 4-1-1 = 2$

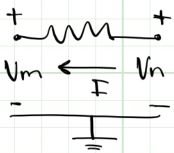
$V_1 = 12\text{ V}$

constrain

2 equat. needed to solve ckt

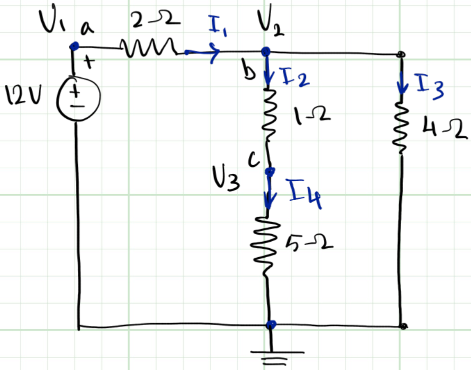
- constrain condition is that V_1 will always equal 12 V.





$I = \dfrac{V_m - V_n}{R}$ $\qquad$ $I = \dfrac{V_n - V_m}{R}$

Node Analysis Eg 1



Find the node Voltages and current through 5Ω resistor and power dissapated by it

$I_2 = I_4$

$N - 1 - N_V = 4 - 1 - 1 = 2$ eq.

Constraint Condition:

$V_I = V_D$

$12 = V_1 \rightarrow \underline{V_1 = 12V}$

b: $I_1 = I_2 + I_3$

$$\frac{V_1 + V_2}{2} = \frac{V_2 - V_3}{1} + \frac{V_2 - 0}{4}$$

$$\frac{12 + V_2}{2} = V_2 - V_3 + \frac{V_2}{4} \quad (1)$$

$$\frac{1}{2}V_1 + \frac{1}{2}V_2 = V_2 - \frac{5}{6}V_2 + \frac{1}{4}V_2$$

$$\frac{1}{2}V_2 - V_2 + \frac{5}{6}V_2 - \frac{1}{4}V_2 = -\frac{1}{2}V_1$$

$$\frac{V_2}{12} = -\frac{1}{2}V_1 \Rightarrow V_2 = -6V_1 = -6(12) = \underline{-72V = V_2}$$

c: $I_2 = I_4$

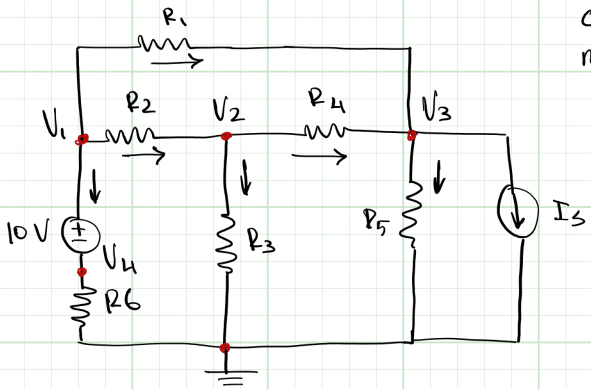
$$V_2 - V_3 = \frac{V_3 - 0}{5}$$

$$V_3 = 5V_2 - 5V_3 \quad (11)$$

$$6V_3 = 5V_2$$

$$V_3 = \frac{5}{6}V_2 = \underline{-60V = V_3}$$

Nodal Ckt 2



Write the equations that can be used to find the node voltages

$N - 1 - N_V = 5 - 1 - 1 = 3$

$VD = VR$

$V_1 = V_4 + 10$ (IV)

Node 1

$$0 = \frac{V_1 - V_3}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_4 - 0}{R_6} \quad (I)$$

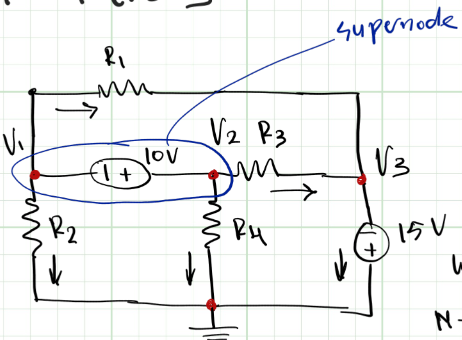
Node 2

$$\frac{V_1 - V_2}{R_2} = \frac{V_2 - 0}{R_3} + \frac{V_2 - V_3}{R_4} \quad (II)$$

Node 3

$$\frac{V_1 - V_3}{R_1} + \frac{V_2 - V_3}{R_4} = \frac{V_3 - 0}{R_5} + I_s \quad (III)$$

Nodal Ckt 3



Supernode
↳ 2 nodes that share same voltage source (none of the nodes are a reference node)

Write equations for node voltages

$N - 1 - N_V = 4 - 1 - 2 = 1$ eq.

Constraint Conditions

$VI = VD$ $PI = PD$

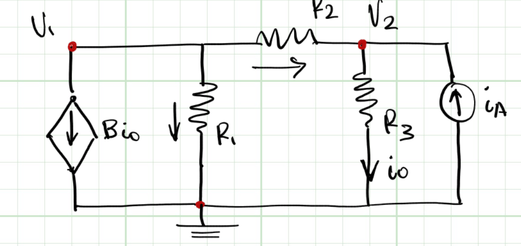
$V_3 + 15 = 0$ $V_1 + 10 = V_2$

$\underline{\underline{V_3 = -15}}$ $\underline{\underline{V_2 = V_1 + 10}}$

Supernode

$$0 = \frac{V_1 - V_3}{R_1} + \frac{V_2 - V_3}{R_3} + \frac{V_2 - 0}{R_4} + \frac{V_1 - 0}{R_2}$$

Nodal CKt 4



For dependent sources, express controlling variable (i_o) in terms of variables of interest (voltages)

$V_2 = i_o R_2 \Rightarrow i_o = \frac{V_2}{R_2}$

Assuming we know R_3, i_A, and B, write down node voltages.

$N-1-N_V = 3-1 = 2$ eq, no constraint cond.

Node 1

$$0 = Bi_o + \frac{V_1 - V_2}{R_2} + \frac{V_1 + 0}{R_1}$$

$$0 = \frac{BV_2}{R_2} + \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_1} \qquad (I)$$

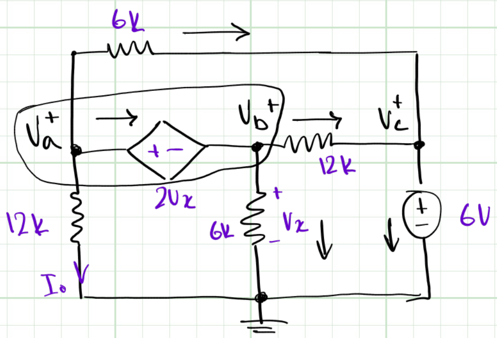
Node 2

$$\frac{V_1 - V_2}{R_2} + i_A = \frac{V_2 - 0}{R_3}$$

$$\frac{V_1 - V_2}{R_2} + i_A = \frac{V_2}{R_3} \qquad (II)$$

Nodal Eg

Find V_a, V_b, I_o



$N - 1 - N_V = 4 - 1 - 2 = 1$

Constraints

$V_b = V_x$

$V_c = 6V$

$V_a = 2V_x + V_b$

$V_a = 2V_b + V_b$

$V_a = 3V_b$

Supernode

$\Sigma E = \Sigma L$

$$0 = \frac{V_b - V_c}{12k} + \frac{V_b}{6k} + \frac{V_a - V_c}{6k} + \frac{V_a}{12k}$$

$$0 = \frac{V_b - 6}{12k} + \frac{V_b}{6k} + \frac{V_a - 6}{6k} + \frac{V_a}{12k}$$

$$0 = \frac{V_b - 6}{12k} + \frac{V_b}{6k} + \frac{3V_b - 6}{6k} + \frac{3V_b}{12k}$$

$$0 = \frac{V_b - 6 + 2V_b + 6V_b - 12 + 3V_b}{12k}$$

$$0 = \frac{12V_b}{12k} \frac{-18}{12k} \Longrightarrow V_b = \frac{18}{12} = \frac{3}{2}V = V_b$$

$$V_a = 3\left(\frac{3}{2}\right) = \frac{9}{2}V$$

$$V_a = \frac{9}{2}V$$

$$I_o = \frac{V_a - 0}{12k} = \frac{9/2\ V}{12k\Omega}$$

$$I_o = 3.75 \times 10^{-4} A$$

$$I_o = 0.375 mA$$

Mesh Analysis

- Law used is KVL.
- Variable of interest is current in a loop/mesh.
- Number of equations:

$$\boxed{\#\text{ indep. loops} - N_I = B - N + 1 - N_I}$$

\# of equations needed to solve mesh analysis ckt

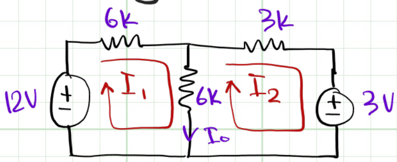
\# of branches

\# of nodes

\# of indep or dep current sources

$V = IR$

Mesh Eg 1



Loop 1

$(I_1 - I_2)6k$

Loop 2

$(I_2 - I_1)6k$

$\underbrace{(I_1 - I_2)6k \qquad (I_2 - I_1)6k}_{\text{for middle resistor}}$

- plus at tail of arrow
- resistors → potential drop

Find I_o in Ckt.

For I_o, since it points in same direction as I_1,

$I_o = I_1 - I_2$

Loop 1

PI = PD

$12V = I_1 \cdot 6k + (I_1 - I_2) \cdot 6k \Rightarrow$

$12 = 6000 I_1 + 6000(I_1 - I_2)$

$12 = 6000 I_1 + 6000 I_1 - 6000 I_2$

$12 = 12000 I_1 - 6000 I_2$

$+2(-3 = -6000 I_1 + 9000 I_2)$

$6 = 12000 I_2$

$I_2 = 5 \times 10^{-4}$ A

Loop 2

PI = PD

$0 = I_2 \cdot 3k + (I_2 - I_1) \cdot 6k + 3$

$-3 = 3000 I_2 + 6000(I_2 - I_1)$

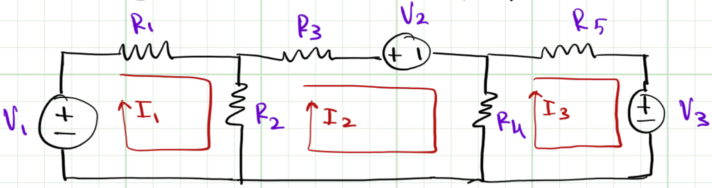
$-3 = 3000 I_2 + 6000 I_2 - 6000 I_1$

$-3 = 9000 I_2 - 6000 I_1$

$I_1 = (3 - 9000(5 \times 10^{-4})) \div -6000 =$

Mesh Eg 2

Write loop equ. that solves circuit



$B - N + 1 - N_i = 8 - 6 + 1 - 0 = 3$ equ.

Loop 1

PI = PD

$V_1 = I_1 R_1 + (I_1 - I_2) R_2$ ✓

Loop 2

PI = PD

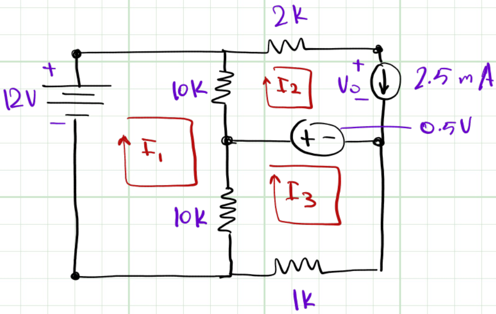
$0 = I_2 R_3 + V_2 + (I_2 - I_3) R_4 + (I_2 - I_1) R_2$ ✓

Loop 3

PI = PD

$0 = I_3 R_5 + V_3 + (I_3 - I_2) R_4$ ✓

Mesh Eg 3



Find V_o

$B - N + 1 - Ni = 7 - 5 + 1 - 1$
$= 2$ eq

Constraint

$I_2 = 2.5\text{ mA}$

because current source ensures this is always true

Loop 1

$PI = PD$

$12V = (I_1 - I_2)\cdot 10k + (I_1 - I_3)\cdot 10k \qquad (\mathrm{I})$ ✓

Loop 2

$PI = PD$

$0.5 = I_2 \cdot 2k + V_o + (I_2 - I_1)\cdot 10k \qquad (\mathrm{II})$ ✓

Loop 3

$PI = PD$

$0 = 0.5 + I_3 \cdot 1k + (I_3 - I_1)\cdot 10k \qquad (\mathrm{III})$ ✓

$(\mathrm{I})\quad 12 = 10000 I_1 - 10000 I_2 + 10000 I_1 - 10000 I_3$

$0 = 20000 I_1 - 10000 I_3 - 10000(2.5 \times 10^{-3})$

$-25 = 20000 I_1 - 10000 I_3$

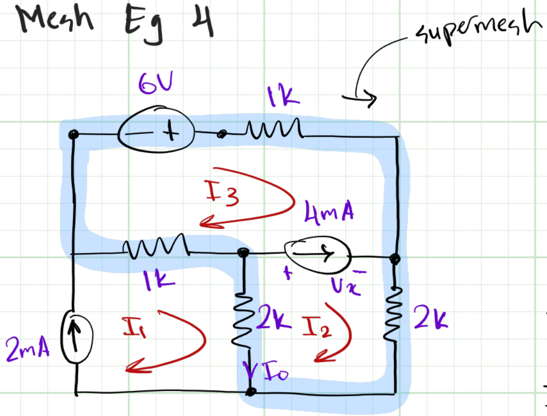
$(\mathrm{III})\quad 0 = 0.5 + 1000 I_3 + 10000 I_3 - 10000 I_1$

$-0.5 = -10000 I_1 + 11000 I_3$

$$\begin{bmatrix} 20000 & -10000 & -25 \\ -10000 & 11000 & -0.5 \end{bmatrix} \sim \begin{bmatrix} 0 & 12000 & -26 \\ -10000 & 11000 & -0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & -2.167 \times 10^{-3} \\ 1 & -1.1 & 5 \times 10^{-4} \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & -2.167 \times 10^{-3} \\ 1 & 0 & -1.8837 \times 10^{-3} \end{bmatrix}$$

Mesh Eg 4



Find I_o

2 meshes share a current source, you have a supermesh

$B - N + 1 - N_i = 7 - 5 + 1 - 2 = 1$

Constraints

$I_1 = 2mA$ — loop 1

$I_2 - I_3 = 4mA$ — loop 3

$\hookrightarrow I_2 = I_3 + 4$

$= -\frac{2}{3} + 4 = \frac{10}{3}$ mA

KVL Supermesh

$PI = PD$

$(6V = (I_2 \cdot 2k) + (I_2 - I_1) \cdot 2k + (I_3 - I_1) \cdot 1k + I_3 \cdot 1k) \div 1 \times 10^{-3}$

$6mV = 2I_2 + (I_2 - 2)2 + (I_3 - 2) + I_3$ (I in mA)

$6 = 2(I_3 + 4) + (I_3 + 4 - 2)2 + I_3 - 2 + I_3$

$6 = 2I_3 + 8 + 2I_3 + 4 + I_3 - 2 + I_3$

$6 - 8 - 4 + 2 = 6I_3$

$I_3 = -\frac{2}{3}$ mA

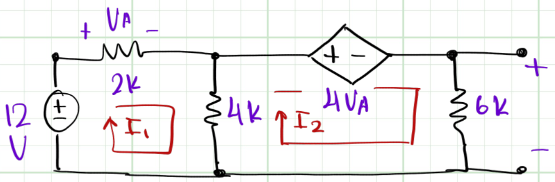
$\therefore I_o = -\frac{4}{3}$ mA

$I_o = I_1 - I_2$

$= 2 - \frac{10}{3} = -\frac{4}{3}$ mA

Mesh Ckt With Dep. Source

Find value of V_o



$B - N + 1 - N_i = 5 - 4 + 1 - 0 = 2$ eq.

Write the controlling Variable in terms of Variables of interest (mesh currents)

$V_A = I_1 \times 2k = 2I_1$

$[mV], [mA], [\Omega]$

Loop 1

$PI = PD$

$$\frac{12\,V}{1\times10^{-3}} = \frac{I_1 \times 2\cancel{k} + (I_1 - I_2)\times 4\cancel{k}}{1\times10^{-3}} +$$

$V \to mV$
$I \to mA$
$k\Omega \to \Omega$

$12 = 2I_1 + 4(I_1 - I_2)$

$\hookrightarrow 12 = 2I_1 + 4I_1 - 4I_2$

$12 = 6I_1 - 4I_2 \quad (I)$

Loop 2

$PI = PD$

$0 = 4V_A + 6I_2 + 4(I_2 - I_1)$

$0 = 8I_1 + 6I_2 + 4I_2 - 4I_1$

$0 = 4I_1 + 10I_2 \quad (II)$

$V_o = I_2 6$

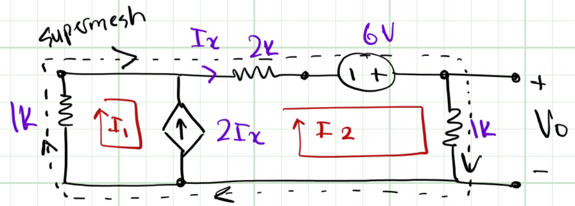
$= -\frac{12}{19} \times 6 = -\frac{72}{19}\ V$

$$\begin{bmatrix} 6 & -4 & 12 \\ 4 & 10 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -\frac{2}{3} & 2 \\ \frac{4}{10} & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{3} & 2 \\ 0 & \frac{19}{15} & -\frac{4}{5} \end{bmatrix} \sim \begin{bmatrix} 1 & -\frac{2}{3} & 2 \\ 0 & 1 & -\frac{12}{19} \end{bmatrix}$$

$\therefore V_o = -3.8\ [V]$

Mesh Ckt

Find V_o



+ supermesh
• dependent source

$B - N + 1 - N_i = 5 - 4 + 1 - 1 = 1$ eq.

$I_2 - I_1 = 2I_x = 2I_2 \Longrightarrow I_2 - I_1 = 2I_2$ (constraint)

$I_1 = -I_2$

$= -3\,mA$

Supermesh KVL

$PI = PD$

$6mV = 2I_2 + I_2 + I_1$

$6 = 3I_2 - I_2 = 2I_2$

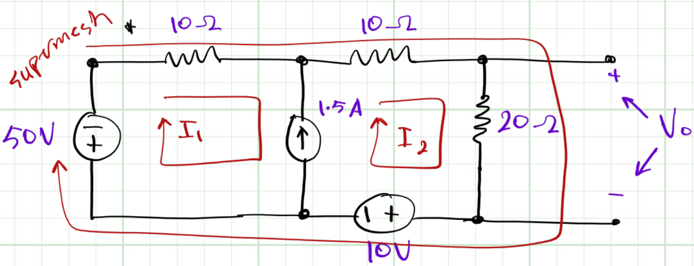
$I_2 = 3\ mA$

$V_o = I_2 \times 1k\Omega$

$= (3\times10^{-3}) \times 1\times10^{3}$

$V_o = 3\ V$

Mesh and Node Ckt



$B - N + 1 - N_i = 6 - 5 + 1 - 1 = 1$ eq.

constraint: $I_2 - I_1 = 1.5\,A \Longrightarrow I_1 = I_2 - 1.5$

Supermesh KVL

$PI = PD$

$$0 = 10I_1 + 10I_2 + 20I_2 + 10 + 50$$

$$-60 = 10(I_2 - 1.5) + 30I_2$$

$$-60 = 40I_2 - 15$$

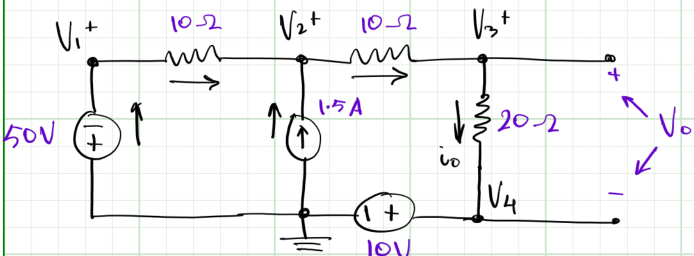
$$40I_2 = -45$$

$$I_2 = -\frac{9}{8}$$

$$V_o = 20I_2$$

$$= -\frac{9 \times 20}{8} = -22.5$$

$$\therefore V_o = -22.5\ V$$



$$N - 1 - N_V = 5 - 1 - 2 = 2\ eq$$

$$V_o = i_o R$$

constraints: $V_1 = -50$

$V_4 = 10V$

KCL at node 2

$CE = CL$

$$\frac{V_1 - V_2}{10} + 1.5 = \frac{V_2 - V_3}{10}$$

$$\frac{-50 - V_2}{10} + 1.5 = \frac{V_2 - V_3}{10}$$

$$-50 - V_2 + 15 = 10V_2 - 10V_3$$

$$-35 = 11V_2 - 10V_3$$

$$V_2 = \frac{-35 + 10V_3}{11}$$

$$10V_2 - 10V_3 = V_3 - 10$$

$$10V_2 + 10 = 11V_3$$

$$V_3 = \frac{1}{11} \cdot 10\left(\frac{-35 + 10V_3}{11}\right) + 10$$

$$V_3 = \frac{10}{11} \cdot \frac{-35}{11} + \frac{10}{11} \cdot \frac{10V_3}{11} + 10$$

$$V_3 - \frac{100}{121}V_3 = \frac{-350}{121} + 10$$

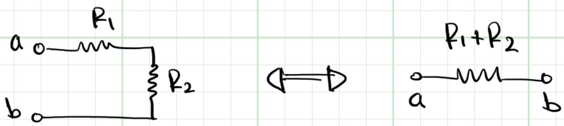
$$V_3 = \left(\frac{-350}{121} + 10\right) \div \left(1 - \frac{100}{121}\right) =$$

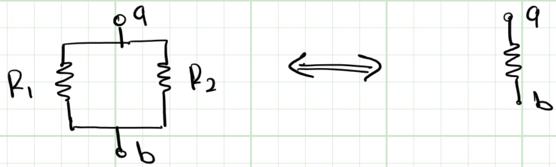
KCL @ node 3

$$\frac{V_2 - V_3}{10} = \frac{V_3 - V_4}{20}$$

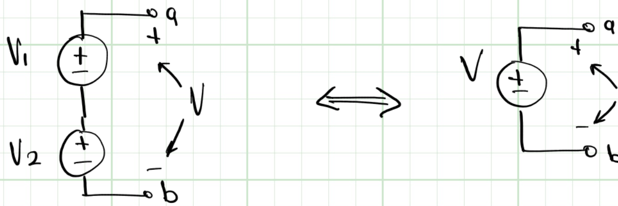
$$\frac{V_2 - V_3}{10} = \frac{V_3 - 10}{20}$$

Equivalence

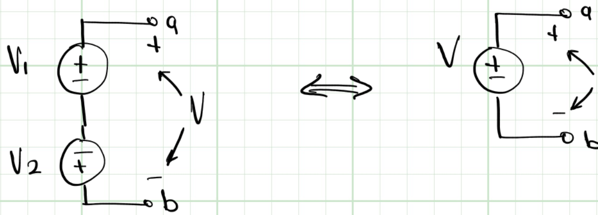




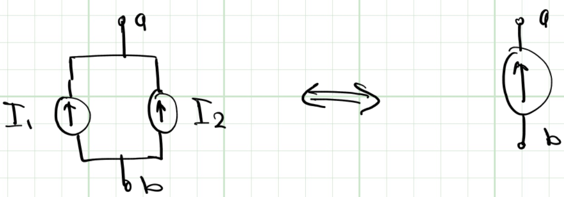
$\frac{R_1 R_2}{R_1+R_2}$



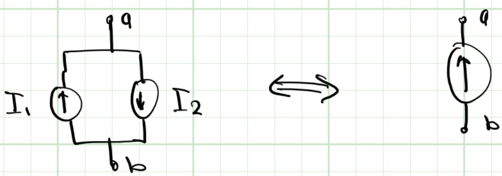
$V = V_1 + V_2$



$V = V_1 - V_2$

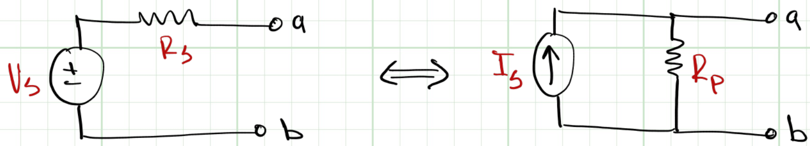


$I = I_1 + I_2$



$I = I_1 - I_2$

Source Transform



if $\boxed{V_s = I_s R_s}$ $\boxed{R_s = R_p = R}$

- disaple voltage by
- disable current by

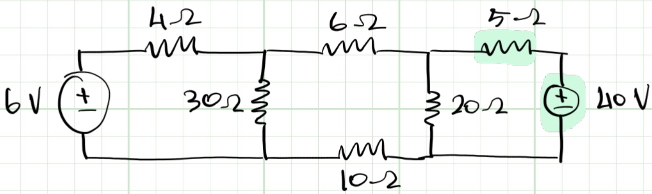
Week 8 : Lec 1

Source Transform Ex

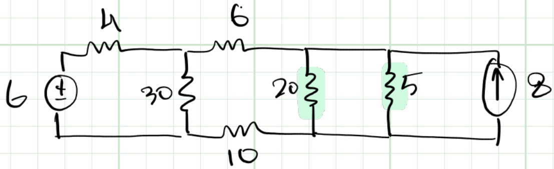
Find pwr 6V battery

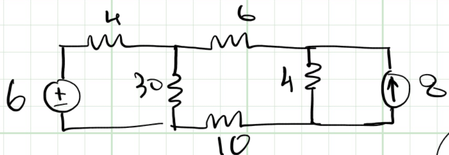
- repeatedly apply source transform to simplify ckt.
- switch between parallel + current src and series + Voltag src

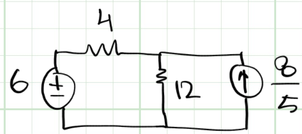
$$\boxed{V_s = R_s I_s} \qquad \boxed{R_p = R_s}$$

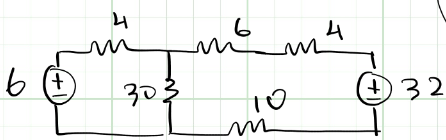


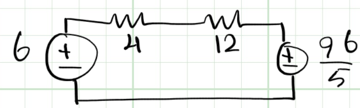
Use source transform to find power in 12 V battery.

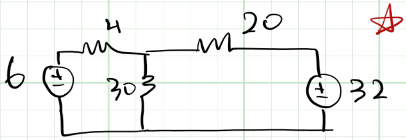


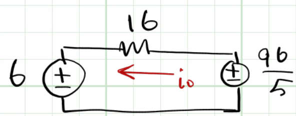


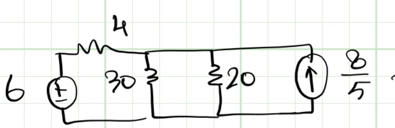










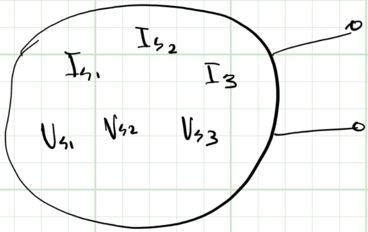


$$i_o = \frac{\frac{96}{5} - 6}{16} = 0.825\,A$$

$$P = i_o \cdot V = (0.825)(6)$$

$$P = 4.95 \ W$$

Linearity



$$V_{out} = \alpha_1 V_{s_1} + \alpha_2 V_{s_2} + \alpha_3 V_{s_3} + \cdots$$
$$+ \beta_1 I_{s_1} + \beta_2 I_{s_2} + \beta_3 I_{s_3}$$

→ Eg: We have ckt with indep. V_s and indep. I_s. We observe:

↳ when $V_s = 10$ and I_s disabled, $V_{out} = 3.5$

↳ when $V_s = 10$ and $I_s = 1$, $V_{out} = 0.5$

Find V_o when $V_s = 4$ [V] and $I_s = 1$ [A]

$$\left.\begin{aligned} 3.5 &= 10V_s + 0I_s \\ 0.5 &= 10V_s + 1I_s \end{aligned}\right\} \quad V_{out} = \frac{7}{20}V_1 - 3I_s$$

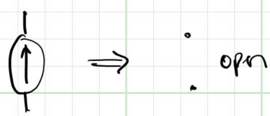
$$V_{out} = \frac{7}{20}(4) - 3(1) = -1.6\,V$$

$\therefore V_{out} = -1.6V$

Superposition

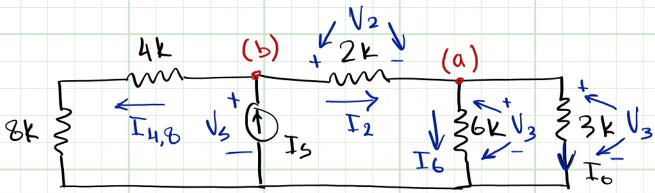
- deactivate voltage source

- deactivate current source



- can use superposition for voltage and current, not power
- careful when de-activating dependent source (better not to do it)

Linearity and Scaling Eg



For this ckt, use linearity to calculate I_o when $I_s = 6\,mA$

- strategy: assume I_o is 1 mA and find I_s, then use linearity property to find I_o when I_s is 6 mA.

① If $I_o = 1mA$ and $R = 3k$

$$V_3 = I_o R = (1mA)(3k\Omega) = 3V = V_3$$

② If $V_3 = 3V$ and $R = 6k$

$$I_6 = \frac{V_3}{R} = \frac{3V}{6k\Omega} = \frac{1}{2}\,mA = I_6$$

③ Do KCL at node (a)

CE = CL

$$I_2 = I_6 + I_o = \frac{1}{2}mA + 1mA = \frac{3}{2}mA = I_2$$

④ If $I_2 = \frac{3}{2}mA$ and $R = 2k\Omega$

$$V_2 = \left(\frac{3}{2}mA\right)(2k\Omega) = 3V$$

Do Ratio

$$\frac{I_s}{I_o} = \frac{2}{1} = \frac{6}{I_{o\,new}}$$

$\therefore I_o = 3\,mA$

⑤ Do KVL to find V_s

$$V_s = V_2 + V_3 = 3 + 3 = 6V = V_s$$

⑥ Find $I_{4,8}$ given $V_s = 6V$ and combined $R = 8+4 = 12\,k\Omega$

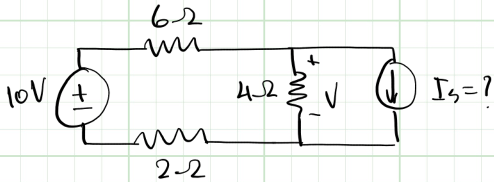
$$I_{4,8} = \frac{V_s}{R} = \frac{6V}{12k\Omega} = \frac{1}{2}mA$$

⑦ Find I_s by KCL @ b

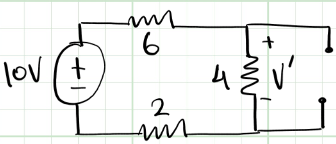
$$I_s = I_{4,8} + I_2 = \frac{1}{2}mA + \frac{3}{2}mA = \boxed{2mA}$$

Superposition Eg 1

What should be value of I_s if V (across 4Ω resistor) is zero. Use superposition

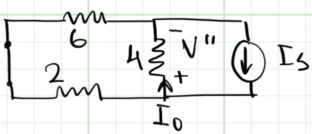


→ First, disable the current source and find V'



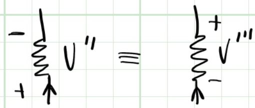
$$V' = 10 \cdot \frac{4}{6+2+4} = \frac{10}{3}\,[V]$$

→ Next, disable the voltage source and find V'' and V'''



$4 \parallel (6 \leftrightarrow 2)$

$4 \parallel 8$



$$I_o = I_s \cdot \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{8}} = \frac{2}{3} I_s\,[A]$$

$$V'' = 4 I_o = \frac{8}{3} I_s\,[V]$$

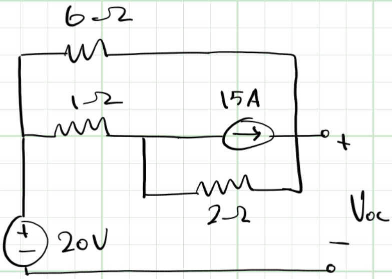
$$V''' = -V'' = -\frac{8}{3} I_s\,[V]$$

$$V = V' + V''' = \frac{10}{3} - \frac{8}{3} I_s = 0$$

$$\frac{10}{3} = \frac{8}{3} I_s \Rightarrow I_s = \frac{10}{8} = \frac{5}{4}\,[A] = I_s$$

Superposition Eg 2

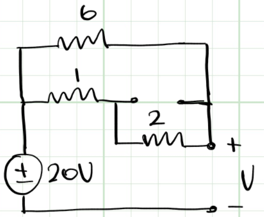
Use superposition to find V_{oc}



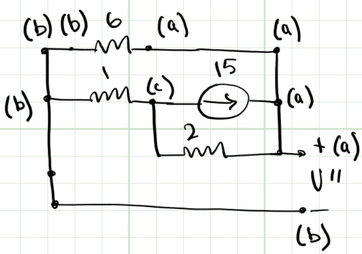
→ First disable I_s (get V')

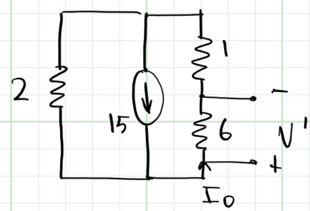
→ Second disable V_s (get V'')

→ $V_{oc} = V' + V''$



$V' = 20$ [V]





$2 \parallel (1 \leftrightarrow 6)$

$2 \parallel 7$

$$I_o = 15 \cdot \frac{\frac{1}{7}}{\frac{1}{7} + \frac{1}{2}} = \frac{10}{3} \text{ [A]}$$

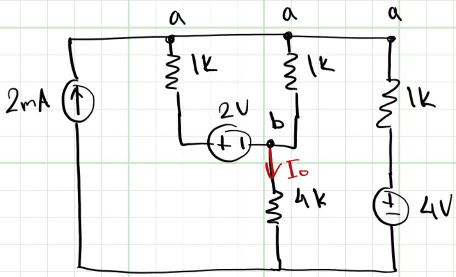
$V' = 6 I_o = 20$ [V]

$V_{oc} = V' + V'' = 20 + 20 = 40$ [V]

$\therefore V_{oc} = 40$ [V]

Superposition Eg 3

Use superposition to find I_o



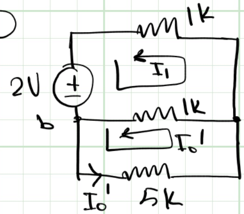
→ Find I_o' with 2V alone

→ Find I_o'' with 4V alone

→ Find I_o''' with 2mA alone

1k

(1)



a b c d

$1k \parallel 1k \parallel (4k \leftrightarrow 1k)$

$1k \parallel 1k \parallel 5k$

$0 = (I_1 - I_o')1k + 1kI_1 + 2$

$0 = 1kI_1 - 1kI_o' + 1kI_1 + 2$

$-2 = -1kI_o' + 2kI_1$ (eq. 1)

$0 = 5kI_o' + (I_o' - I_1)1k$

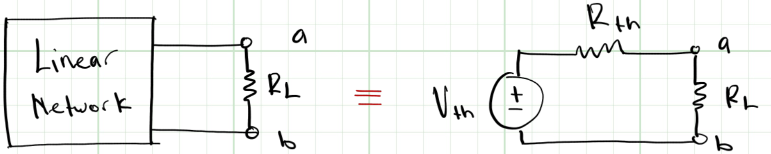
$0 = 5kI_o' + 1kI_o' - 1kI_o$

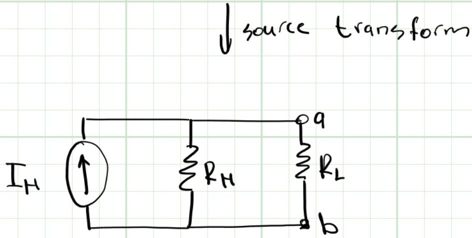
$0 = 6kI_o' - 1kI_o$ (eq. 2)

$I_1 = -1/5500$

$I_o' = -3/2750$

Thevenin and Norton Equivalent CKt





$V_{Th} = I_N R_{Th}$

and $R_{th} = R_N$

How to find V_{th}

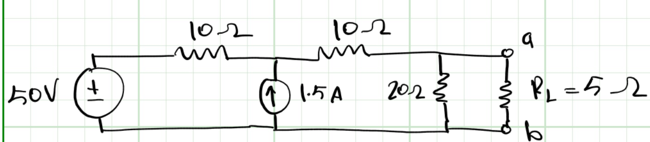
* Go to terminal of interest, open it, and measure the open ckt voltage

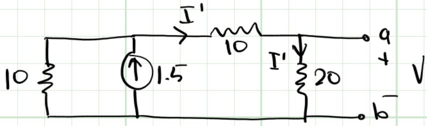
How to find R_{th}

* Go to terminal of interest, short it, calculate short ckt current, and find $R_{th} = \dfrac{V_{th}}{i_{shckt}}$

Thevenin Eg 1

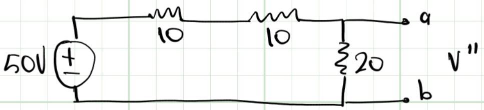
Find V_{Th} wrt terminals a-b where $R_L = 5\,\Omega$ and also R_{Th}





$$I' = 1.5 \cdot \frac{\frac{1}{30}}{\frac{1}{10} + \frac{1}{30}} = \frac{3}{8}\ [A]$$

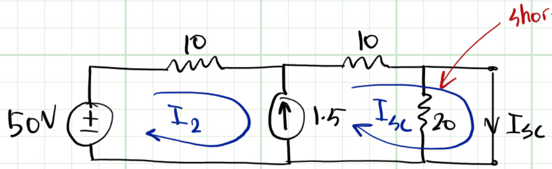
$$V' = 20 I' = \frac{15}{2}\ [V]$$



$$V'' = 50 \cdot \frac{20}{10+10+20} = 25 \text{ [V]}$$

$$\therefore V_{oc} = V' + V'' = \frac{15}{2} + 25 = \frac{65}{2} = 32.5 \text{ [V]}$$

$$\therefore V_{oc} = V_{Th} = 32.5 \text{ [V]}$$



<u>Supermesh KVL</u> $\quad 1.5 = I_{sc} - I_2 \quad$ (eq 1)

$$50 = 10 I_2 + 10 I_{sc} \quad (\text{eq } 2)$$

$I_{sc} = 13/4$ [A]

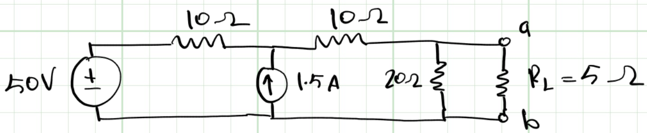
$I_2 = 7/4$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{32.5}{3.25} = 10\ \Omega$$

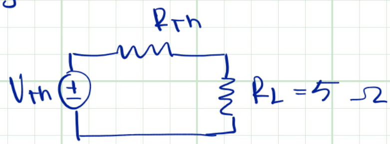
$$\therefore R_{Th} = 10\ \Omega$$

Thevenin Eg 1

see last lecture note for full solution



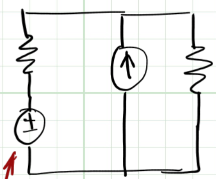
change into



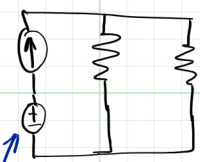
Formula

- if only have independent sources
- shortcut with R_{Th}
  - ↳ 1) deactivate all sources (voltage and current)
    2) look back and ask "what is equivalent resistance of remaining things)
    3) $R_{eq} = R_{th}$

Note



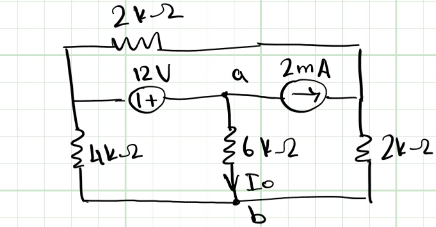
not a current divider because Voltage src is on diff Branch

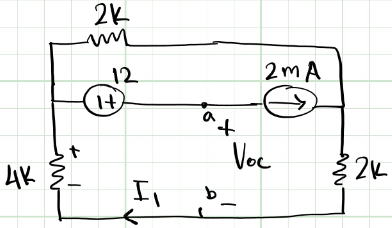


Yes, current divider because V_s on same branch as I_s

Thevenin Eg: 2

Use the Thevenin equivalent ckt to find the I_o



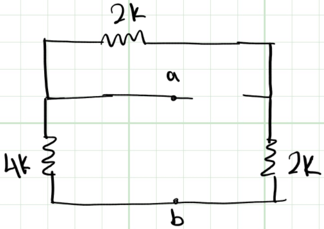


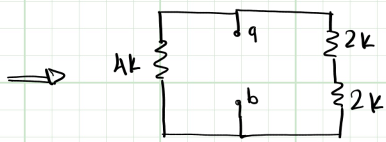
$$I_1 = 2(10^{-3}) \cdot \frac{\frac{1}{6\times10^3}}{\frac{1}{2\times10^3} + \frac{1}{6\times10^3}} = 5\times10^{-4} \text{ [A]}$$

KVL : PI = PD

$$V_{oc} = 12 + 4k(-5\times10^{-4}) = 10 \text{ [V]}$$

$\therefore V_{Th} = 10$ [V]

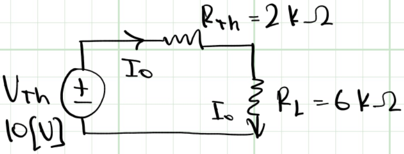




$$4k \parallel (2k \leftrightarrow 2k) = \left(\frac{1}{4k} + \frac{1}{2k+2k}\right)^{-1}$$

$\therefore R_{eq} = R_{Th} = 2000\ \Omega$

$\therefore R_{Th} = 2\ k\Omega$

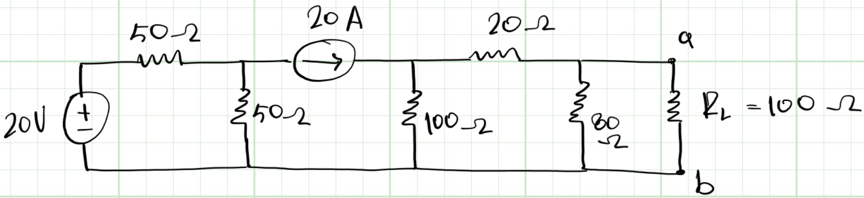


$$V_L = 10 \cdot \frac{6k}{2k+6k} = \frac{15}{2} \text{ [V]}$$

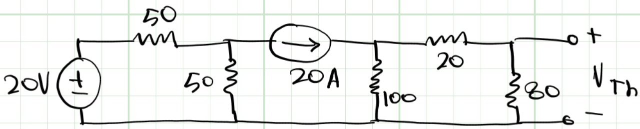
$$I_o = \frac{V_L}{R_L} = \frac{15/2}{6k} = 1.25\times10^{-3} \text{ [A]}$$

$\therefore I_o = 1.25$ mA

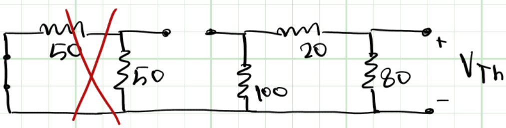
Thevenin Eg 3



Find the Thevenin Equivalent Ckt for the ckt at terminal a-b and find power dissipated by R_L.



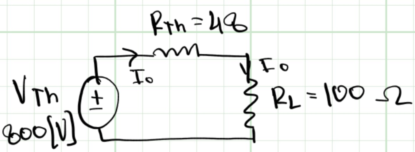
$$V_{Th} = 10[A] \times 80\ [\Omega] = \underline{\underline{800\ [V] = V_{Th}}}$$



$$\therefore R_{Th} = 48\ [\Omega]$$

$$R_{Th} = R_{eq} : \ (100 \leftrightarrow 20) \parallel 80$$

$$\left(\frac{1}{120} + \frac{1}{80}\right)^{-1} = 48\ [\Omega]$$



$$V_{Th} = I_o R_{eq}$$

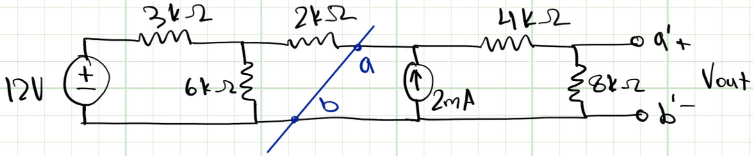
$$I_o = \frac{800}{48+100} = \frac{200}{37}$$

$$P = I_o^2 R = \left(\frac{200}{37}\right)^2 (100)$$

$$\therefore P = 2921.84\ [W]$$

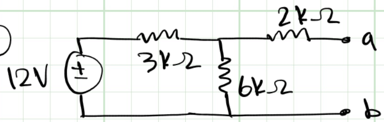
Thevenin Breaking ckt

* when breaking a ckt with dependent source, make sure you keep dependent src and its controlling parameter on same side.



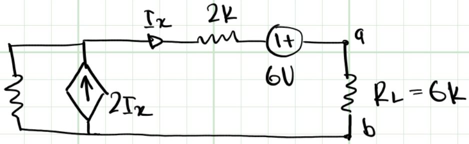
Use Thevenin twice to find Vout (break the ckt at a-b)

①

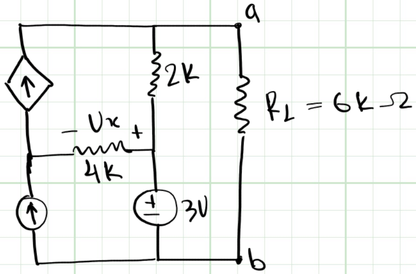


Week 9: Lec 1

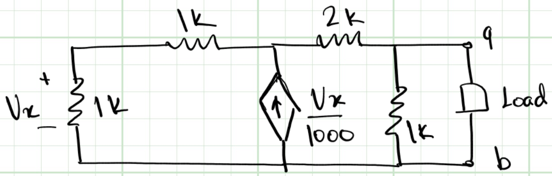
Thevenin Ckts : Dependent Sources



Thevenin Ckt Dependent



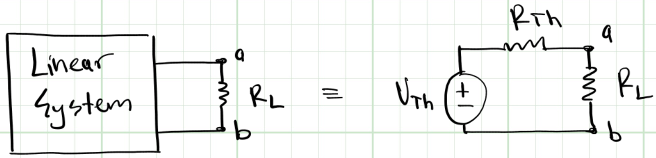
Thevenin Eg 3 V_{TH} is zero



- when you have no dependent src, $V_{TH} = 0$.
- To find R_{TH}, attach battery and

Maximum Power Transfer

- If load has R_{Th}, then maximum power is transferred



For max power transfer, set $\boxed{R_L = R_{Th}}$

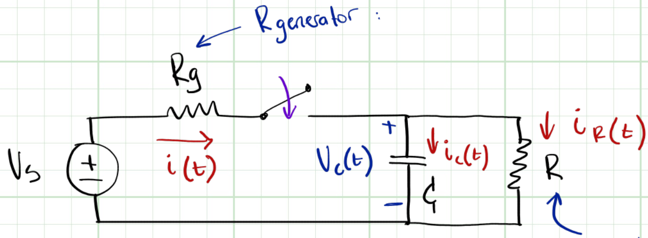
$$\boxed{\text{Max Power} = \frac{(V_{Th})^2}{4R_{Th}} = \frac{(V_{Th})^2}{4R_L}}$$

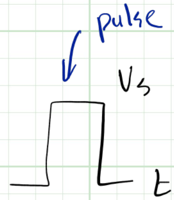
Week 9: lec 2

- when calculating R_{Th}, it must be (+).
  - ↳ let's you check if V_{Th} and I_{sc} are correct
- If ckt has dependent srcs, break it and find Thevenin ckt
  - ↳ when you attatch it back, your new ckt has no more dependent ckt → you can easily find R_{Th}/R_L.
- For good Thevenin breaking ckt check Lec 29 Examples

Week 9: Lec 3

Transient for 1st Order Ckt





capacitors have a short resistor

- transient : time dependency
- source is still a battery (DC)
- 1st Order because 1 storage element (one type, like inductor or capacitor)
  - ↳ if 2 of them same kind → can combine

- doesn't happen instantaneously
- takes

- Every battery : resistor
- Every capacitor :
- Every inductor :

→ A $t = t_0$ → we close switch

what is $V_c(t) = ?$

$$i(t) = i_c + i_R$$

not function of time

$$\frac{V_s - V_c}{R_g} = C\frac{dV_c(t)}{dt} + \frac{V_c(t)}{R}$$

$$\boxed{C\frac{R_g \cdot R}{R_g + R} \cdot \frac{dV_c(t)}{dt} + V_c(t) = V_s \cdot \frac{R}{R + R_g}}$$

differential equation

- 1st order linear nonhomogeneous ODE

General Form

$x(\infty) = x(t \to \infty)$
steady state

$$\boxed{\tau \frac{d\,x(t)}{dt} + x(t) = x(\infty)}$$

time constant ← controls how fast steady state is reached

$C \cdot (R_g \| R)$

Solution Method

→ Guess $x(t) = k_1 + k_2 e^{-t/\tau}$

→ $\tau\left(-\frac{1}{\tau}\right) k_2 e^{-t/\tau} + k_1 + k_2 e^{-t/\tau} = x(\infty)$

$-k_2 e^{-t/\tau} + k_1 + k_2 e^{-t/\tau} = x(\infty)$

$\therefore \underline{\underline{x(\infty) = k_1}}$

→ given $x(t_0) = x(t=0) = x(0)$

$x(0) = k_1 + k_2 e^{-t/\tau}$

$x(0) = k_1 + k_2$

$x(0) = x(\infty) + k_2$

$\therefore \underline{\underline{k_2 = x(0) - x(\infty)}}$

0^+: tiny instant after switch close

Full Solution :

$$\boxed{x(t) = \underbrace{x(\infty)}_{k_1} + \underbrace{\left[x(0^+) - x(\infty)\right]}_{k_2} e^{-\frac{t}{\tau}}}$$

At different time instead of $t=0$

$$x(t_0^+) = k_1 + k_2 e^{-t/\tau}$$

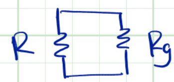
$$\Rightarrow \quad \underline{k_2 = \left[x(t_0^+) - x(\infty)\right] e^{t_0^+/\tau}}$$

delayed time

Full solution : $$\boxed{x(t) = x(\infty) + \left[x(t_0^+) - x(\infty)\right] e^{-(t-t_0)/\tau}}$$

$$\therefore \; V_c(t) = V_s \frac{R}{R+R_g} + \left[0 - \frac{R}{R+R_g}\right] e^{-\frac{t}{\tau}}$$

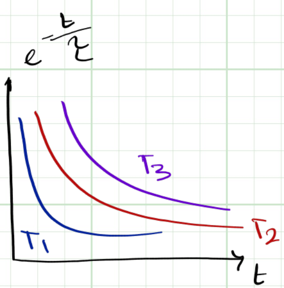
$$\tau = C \cdot \frac{R_g R}{R_g + R} = C\,(R \parallel R_g)$$



as $t \to \infty$, the $e^{-\frac{t}{\tau}}$ term goes to zero

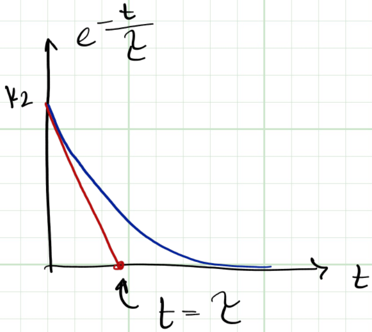
so $V_c(t) = V_s \cdot \frac{R}{R+R_g}$

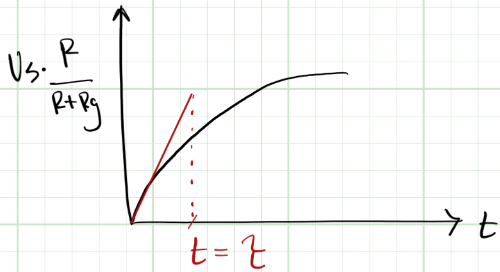
meaning it makes sense because the C is like an open ckt ... voltage divider equ. matches



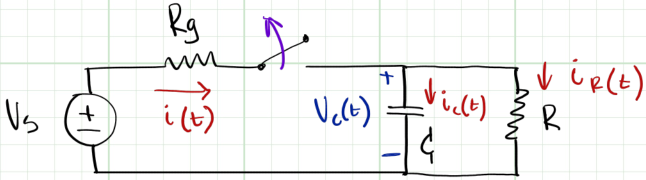
$\tau_1 > \tau_2 > \tau_3$

+ larger the τ, quicker the charging





Now, We Open Switch at $t=t_0$



$$i_c(t) + i_R(t) = 0$$

$$C\frac{dV_c(t)}{dt} + \frac{V_c(t)}{R} = 0$$

$\Rightarrow$ canonical form

steady state

$$\boxed{CR\frac{dV_c(t)}{dt} + V_c(t) = 0}$$

$\therefore V(\infty) = 0$

$\tau = RC$

initial condition

$V_c(t_0^+) = V_s\frac{R}{R+R_g}$

$\therefore V_c(t_0^+) = V_c(t_0^-)$

<u>discharging</u>

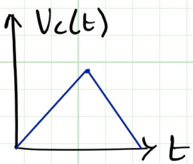
$$V_c(t) = 0 + \left[V \cdot \frac{R}{R+R_g} - 0\right]e^{-t/RC}$$

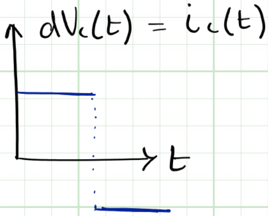
$V_c(t) =$

Notes

- The voltage accross capacitors must be continuous

$$i_c = C \frac{dV_c(t)}{dt}$$

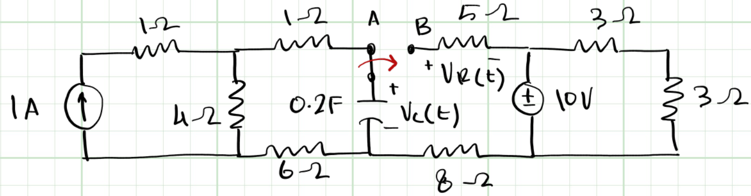




- The current through inductors must be continuous

Step-by-Step Method

$i_C = C\,\frac{dV}{dt}$



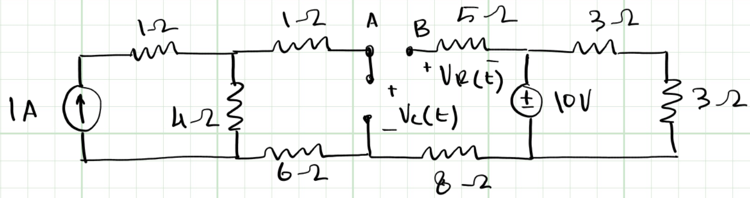
$$x(t) = x(\infty) + [x(0^+) - x(\infty)]\,e^{-\frac{t}{\tau}}$$

$$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]\,e^{-\frac{t}{\tau}}$$

$$V_R(t) = V_R(\infty) + [V_R(0^+) - V_R(\infty)]\,e^{-\frac{t}{\tau}}$$

Find $V_c(0^-)$

$-\infty < t < 0^-$



$\Rightarrow V_c(0^+) = V_c(0^-) = (1\,[A])(4\,[\Omega]) = 4V = V_c(0^+)$

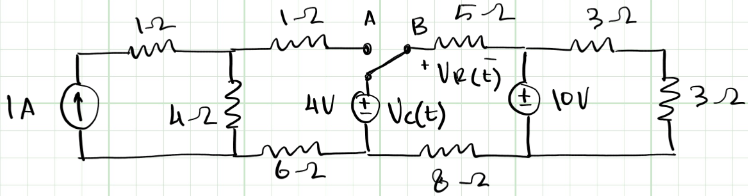
Find $V_R(0^-)$

$\Rightarrow$ for resistor, $V_R(0^+) \neq V_R(0^-)$ bc. $V_R = i_R R$ ← can be discontinuous

$V_R(0^-) = (0\,[A])(5\,[\Omega]) = \underline{0\,V = V_R(0^-)}$

Finding $V_R(0^+)$: During switching, we can replace capacitor with a voltage src of voltage $V_c(0^+) = V_c(0^-)$

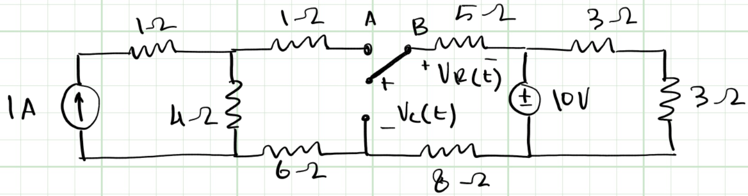
Note: for inductors, replace with current src.



Do any method to find V_R

↳ find $V_R(0^-) = -2.3$ [V]

Find $V_c(\infty)$ and $V_R(\infty)$



$\therefore V_c(\infty) = 10$ V

$V_R(\infty) = 0$ V

Findings

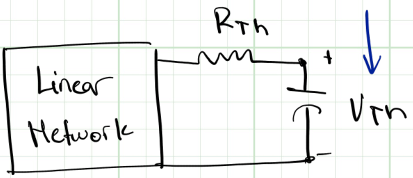
For capacitor: $V_c(0^+) = V_c(0^-) = 4$ V

$V_c(\infty) = 10$ V

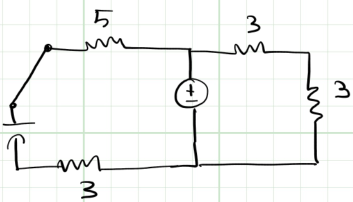
For

Find Time Constant

terminal is where the capacitor was after switch is flipped

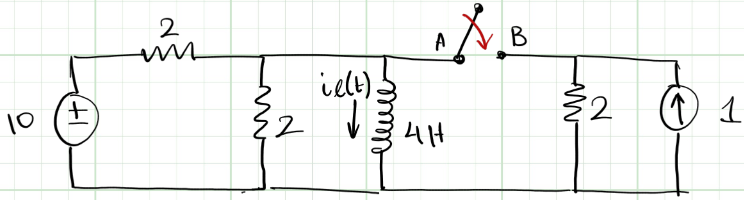


capacitive ckt: $\boxed{\tau = C R_{Th}}$



Inductor CKt

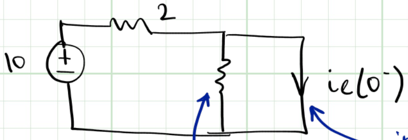
$V_c = L \frac{di_L(t)}{dt}$



$i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)] e^{-\frac{t}{\tau}}$

$\rightarrow i_L(0^-) = i_L(0^+) = 5 [A]$

$i_L(0^-) = \frac{10}{2} = 5 [A]$



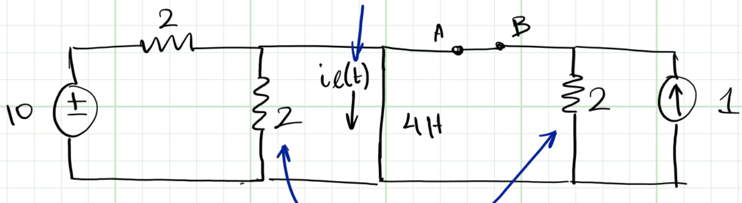
inductor is a short to DC

gets shorted

Find $i_L(\infty)$

- looks like short

Find this



shorted

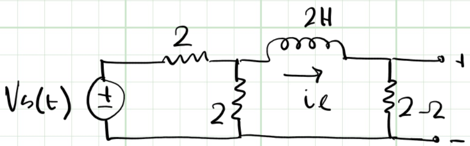
Use nodal: $\longrightarrow$ get $i_L(\infty) = 6 [A]$

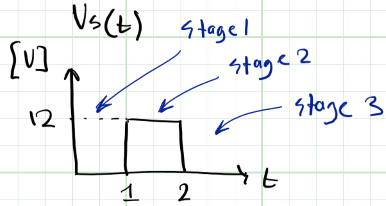
- For inductors : $\boxed{\tau = \frac{L}{R_{Th}}}$

$\therefore i_L(t) = 6 + [5-6] e^{-t/6} = 6 - e^{-t/6}$

Week 10 : Lec 2

Inductor Eg





Find $V_o(t)$ for $-\infty < t < 1$
$1 < t < 2$
$2 < t < \infty$

Stage 0 : $\boxed{V_o(t) = 0\ [V] \quad t \in (-\infty, 1)}$

Stage 1 : $V_o(t) = V_o(\infty) + \left[V_o(t=1^+) - V_o(\infty)\right] e^{-t/\tau}$

stage 1

⇒ We know $V_o(t=1^-) = 0\ [V]$, but $V_o = iR$, so the voltage across resistor doesn't have to be cont.

⇒ However, I see that V_o's resistor is fed by i_L, and since i_L has to be continuous → ∴ V_o is continuous

∴ $V_o(t=1^-) = V_o(t=1^+) = 0\ [V]$

⇒ To find $V_o(\infty)$, replace inductor with a short and find V_o in that case → $V_o(\infty) = 4\ [V]$

⇒ $\tau = \dfrac{L}{R_{th}} = \dfrac{3}{2}$

as seen by inductor after event has happened

$$\boxed{V_o(t) = 4\left[1 - e^{-\frac{(t-1)}{3/2}}\right], \quad t \in (1,2)}$$

Stage 2: $V_o(t) = V_o(\infty) + [V_o(t=2^+) - V_o(\infty)] e^{-t/\tau}$

stage 2

$\Rightarrow$ Notice that R_{th} in stage 2 hasn't changed from stage 1

$\therefore$ τ is the same $\rightarrow$ $\tau = \frac{3}{2}$ s

$\Rightarrow$ as $t \to \infty$, $V_o(\infty) \to 0$ because $V_s(t)$ is zero

$\therefore V_o(\infty) = 0$ [V]

$\Rightarrow$ Finding $V_o(t=2^+) = V_o(t=2^-)$ because Voltage across resistor is fed by i_L, which is always continuous

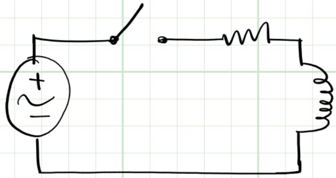
$\Rightarrow$ so we plug in to eq. 2

$$V_o(t=2^-) = 4\left[1 - e^{-\frac{(2-1)}{3/2}}\right] = 3.11 \text{ [V]} = V_o(t=2^+)$$

$$\boxed{\therefore\ V_o(t) = 0 + 3.11\, e^{-\frac{(t-2)}{3/2}} \text{ [V]}, \quad t \in (2, \infty)}$$

Sinusoidal Excitation (AC-Intro)

- needs complex algebra



$$\boxed{x(t) = A\cos(\omega t + \theta)}$$

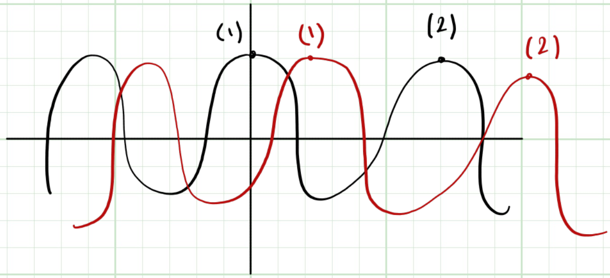
peak amplitude (distance from x-axis to peak)

angular frequency $\left[\frac{rad}{s}\right]$

phase angle [rad] or [degree]

- peak-to-peak amplitude : double of peak amplitude
- period (T) of a function : $\boxed{T = \frac{2\pi}{\omega}}$
- regular frequency $\boxed{2\pi f = \omega}$ or $\boxed{f = \frac{1}{T}}$ [Hz]

Leading & Lagging



1) must have same frequency
2) all must be in sin or all in cos
   - $\sin(\omega t + \phi) = \cos(\omega t + \phi - \pi/2)$
   - $\cos(\omega t + \phi) = \sin(\omega t + \phi + \pi/2)$
3) peak amplitude must be all same sign
   - $-\sin(\omega t + \phi) = \sin(\omega t + \phi - \pi)$
   - $-\cos(\omega t + \phi) = \cos(\omega t + \phi + \pi)$
4) difference must be less than 180° or π rad
   - we can add 360° to argument and nothing changes

Eg: 2

Complex Numbers

$j = \sqrt{-1}$ or $j^2 = -1$

- complex number : $\boxed{z = x + jy}$, $z \in \mathbb{C}$, $x, y \in \mathbb{R}$

  $x = \mathrm{Re}(z)$ $y = \mathrm{Im}(z)$

- complex conjugate: $\boxed{z^* = x - jy}$

- identity : $\boxed{z \cdot z^* = x^2 + y^2 \in \mathbb{R}}$

Complex Exponential and Polar Form

Euler Identity : $\boxed{Ae^{j\alpha} = A\cos\alpha + jA\sin\alpha}$

complex exponential

$$z = x + jy = Ae^{j\alpha} = A\cos\alpha + jA\sin\alpha$$

- $x = A\cos\alpha$
- $y = A\sin\alpha$
- $A = \sqrt{x^2 + y^2}$
- $\alpha = \arctan\left(\frac{y}{x}\right)$

- polar representation : $\boxed{Ae^{j\alpha} = A\angle\alpha}$

Phasors

AC: $V(t) = V_m \cos(\omega t + \theta_v)$

or $V(t) = V_m \sin(\omega t + \theta_v)$

- We will convert from ODE to algebraic equations, but we have to use complex numbers

doesn't depend on t

Let $V(t) = V_m \cos(\omega t + \theta_v)$

$$V(t) = Re\left[V_m e^{j(\omega t + \theta_v)}\right] = Re\left[V_m e^{j\theta_v} \cdot e^{j\omega t}\right]$$

$$= Re\left[\underline{V} e^{j\omega t}\right]$$

where $\boxed{V_m e^{j\theta_v} = \underline{V}}$ is called a phasor

- phasor is the part that does not depend on time

Proof:

real part, no j

$$Re\left[V_m \cos(\omega t + \theta_v) + j \sin(\omega t + \theta_v)\right] = V_m \cos(\omega t + \theta_v)$$

- if given $V(t)$ or $i(t)$ as a sin, convert to cos function and then take real part of it.

Eg: given $i(t) = I_m \sin(\omega t + \theta_i)$

$$i(t) = I_m \cos\left(\omega t + \theta_i - \frac{\pi}{2}\right) = Re\left[I_m e^{j(\omega t + \theta_i - \frac{\pi}{2})}\right]$$

$$= Re\left[I_m \cdot e^{j\omega t} \cdot e^{(\theta_i - \frac{\pi}{2})j}\right]$$

$$\underline{I} = I_m e^{j(\theta_i - \frac{\pi}{2})}$$

so phasor is

Phasor Relationship for RLC

- phasor representation / domain
- frequency domain
- sinusoidal steady state

Resistors : $V_R(t) = i(t)R \Rightarrow \boxed{\underline{V} = R\,\underline{I}}$ $\qquad V_m e^{j\theta_v} = R I_m e^{j\theta_i}$

Inductor : $V_L(t) = L\dfrac{di(t)}{dt} \Rightarrow \boxed{\underline{V_L} = j\omega L\,\underline{I}}$

Capacitor : $i_c(t) = C\dfrac{dV(t)}{dt} \Rightarrow \boxed{\underline{V_c} = \dfrac{1}{j\omega C}\,\underline{I_c}}$

impedences

time domain $\qquad$ phasor domain

- if the real part of 2 phasors are equal, then the 2 phasors are equal.

Week 11 - Lec3

Resistor

→ For $V_m e^{j\theta_v} = R I_m e^{j\theta_i}$ to be true, we know that $V_m = R I_m$ and $\theta_v = \theta_i$

→ For resistors, the voltage and current are in phase

Inductor

→ For $V_m e^{j\theta_v} = j\omega L I_m e^{j\theta_i}$, impedence is $j\omega L$

$\underbrace{j\omega L}$ → unit of $[\Omega]$, so

$$V_m \angle \theta_v = j\omega L I_m \angle \theta_i$$

since $j = e^{j\frac{\pi}{2}} = 1 \angle \pi/2 = 1 \angle 90$

$$V_m \angle \theta_v = \omega L I_m \angle \theta_i + 90^\circ$$

$$\theta_i = \theta_v - 90^\circ$$

→ For inductors, voltage leads current by 90°

Capacitor

$$\frac{1}{j\omega C} = \frac{-j}{\omega C}$$

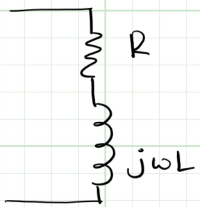
$$V_m e^{j\theta_v} = I_m e^{j\theta_i} \frac{1}{j\omega C}$$

$$\theta_v = \theta_i - 90^\circ$$

→ For capacitors, current leads voltage by 90°

Week 12 : Lec 1

AC Eg 1



$$Z = 5 + j4 \qquad \omega_1 = 1000 \text{ rad/s}$$

$$Z = 5 \qquad \omega_2 = 1300 \text{ rad/s}$$

$$j\omega_1 L = j4$$

$$\omega_1 L = 4$$

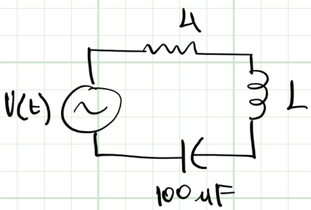
$$L = \frac{4}{\omega_1}$$

$$j\omega_2 L = j\omega_2\left(\frac{4}{\omega_1}\right)$$

$$= j\left(\frac{1300}{1000}\right)4$$

$$= \boxed{}$$

- to make things in same phase, make sure arctan portion is 0.



Week 12 : Lec 2

$A_m \cos(\omega t + \theta) \longrightarrow A_m \angle \theta^\circ$

$A_m \sin(\omega t + \theta) \longrightarrow A_m \cos(\omega t + \theta - \frac{\pi}{2}) \longrightarrow A_m \angle \theta - \frac{\pi}{2}$

Impedance : $\boxed{Z = R + jX}$

impendance, resistance, reactance

G is not $\frac{1}{R}$

Admittance : $\boxed{Y = \frac{1}{Z} = G + jB}$

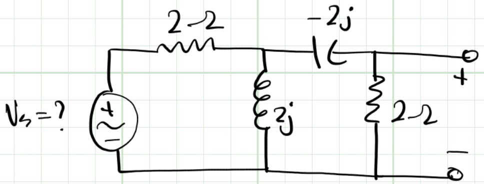
admittance, conductance, susceptance

$\boxed{G = \frac{R}{R^2 + X^2}}$ $\boxed{B = \frac{-X}{R^2 + X^2}}$

$\boxed{R = \frac{G}{G^2 + B^2}}$ $\boxed{X = \frac{-B}{G^2 + B^2}}$

* admittance combine in opposite way to ∥ and ↔ impedances

AC Eg



$V_o = 8 \angle 45^\circ$

Week 12: lec 3

- To solve ckt with both DC and AC sources
  ↳ use superposition by disabling each source and adding them up algebraicly.

- Once you find sum of AC part (in phasor domain)
  ↳ multiply be $e^{j\omega t}$ and take real part (should get cos function)

- At resonance frequence $\omega_0 = \frac{1}{\sqrt{LC}}$, the impedence is purely real (phase angle 0°)

$$\underline{V_s} = \underline{I_s}\, Z$$

$$|V_s| \angle \theta_v = |I_s| \angle \theta_i \; |Z| \angle \theta_z$$

to find a condition that lets V and I be in phase
↳ θ_z must be zero → meaning Z is purely real.

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} \Leftrightarrow$$

$R_{TH} \Leftrightarrow R_{NT}$

Deactivate sources only for all independent circuit.