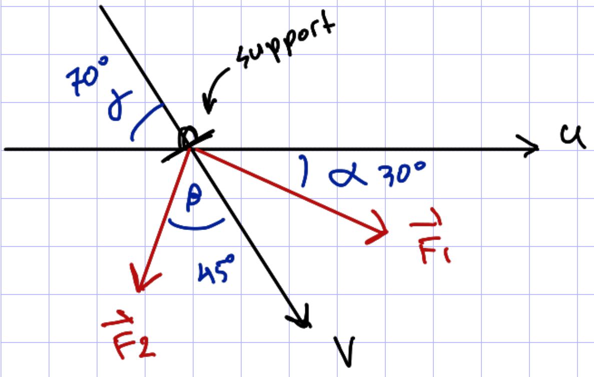
Lecture #1

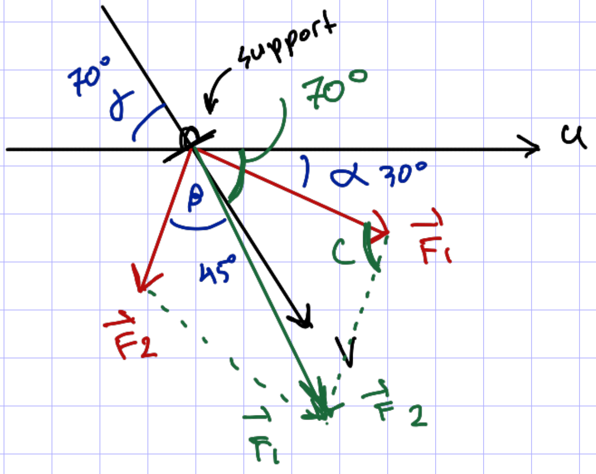
Sep 11, 2023

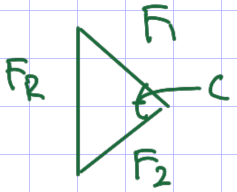
① Eg: Determine the magnitude of the resultant force $\vec{F}_R = \vec{F}_1 + \vec{F}_2$ and its direction measured clockwise from the positive u-axis



Given:
$F_1 = 300$ N
$F_2 = 500$ N
$\alpha = 30°$
$\beta = 45°$
$\gamma = 70°$

Step 1: Apply Parrallelogram
Step 2: Apply cosine law
Step 3: Apply sine law





$$C = 360 - 2(45+40)$$
$$= 95°$$

$$F_R = \sqrt{F_1^2 + F^2 - 2F_1F_2 \cos C}$$
$$=$$
$$F_R = 605.1\text{ N} = \boxed{605\text{ N} = F_R}$$

sig figs - 3 or 4
↳3.5

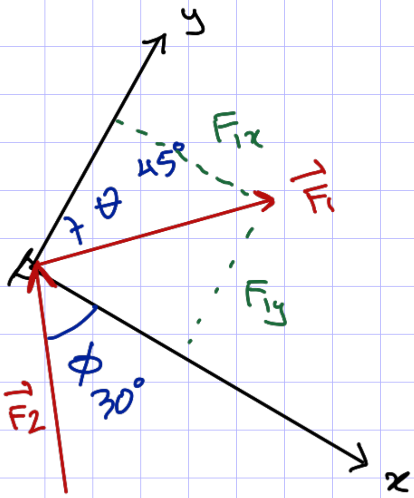
Step 3: $$\frac{F_2}{\sin b} = \frac{F_1}{\sin a} = \frac{F_R}{\sin C}$$

$$\sin b = \frac{F_2 \sin c}{F_R}$$

$$\boxed{b = 55.4°}$$

∴ From u-axis F_r is $30+54°$
$= 85.4 = \theta_{F_R-u}$

② Eg: Determine the x and y components of $\vec{F}_1$ and $\vec{F}_2$.



Given:
$F_1 = 200\,N$
$F_2 = 150\,N$
$\theta = 45°$
$\phi = 30°$

→ Resolve F_1, F_2 in x-y direction

$F_{1x} = {}^{+}F_1 \sin 45° = 141.4\,N$
$F_{1y} = {}^{+}F_1 \cos 45° = 141.4\,N$
} 4 sig figs

$F_{2x} = {}^{-}F_2 \cos 30° = -130.0\,N$ ← 4 sigfigs
$F_{2y} = {}^{+}F_2 \sin 30° = 75.0\,N$ ← 3 sigfigs

Eg: Determine the magnitude and direction measured counterclockwise from x' axis of the resultant force of the 3 forces acting on the bracket

$F_1 = 300$ N

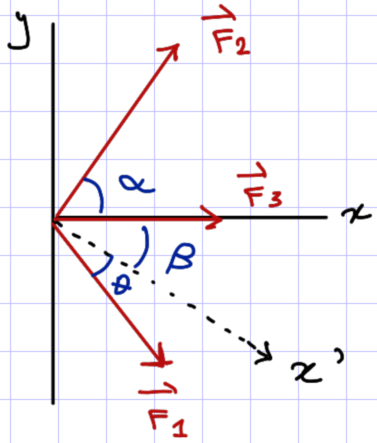
$F_2 = 450$ N

$F_3 = 200$ N

$\alpha = 45°$

$\beta = 30°$

$\theta = 20°$



→ Resolve F_1, F_2, F_3 in x/y direction

→ sum x, y forces to get resultant

→ Calc. angle of F_R CCW from x' axis

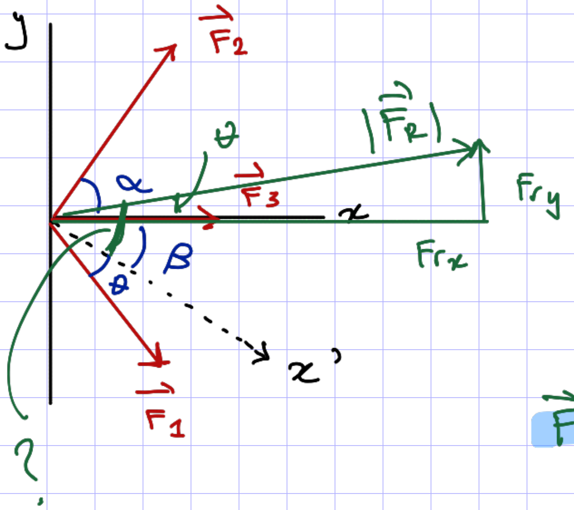
	direct. x-axis	direct y-axis
$F_1 = 300$	$+ 300 \cos 50$	$- 300 \sin 50$
$F_2 = 450$	$+ 450 \cos 45°$	$+ 450 \sin 45°$
$F_3 = 200$	$+ 200$	0

$+$ 711.03 N F_{Rx}

$+$ 88.38 N F_{Ry}

$$|\vec{F_R}| = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{711.03^2 + 88.38^2} = 716.5 \text{ N}$$

≈ 717 N



$$\theta = \tan^{-}\left(\frac{711.03}{88.38}\right) = 7.085$$

$$\text{angle} = \beta + \theta$$
$$= 30° + 7.085°$$
$$= 37.1$$

$\vec{F_R} = 717$ N [37.1° CCW from x' axis]

About Course

- $\Sigma F = 0$ for everything in this course
- start with 2-D
- u-v axes are not orthogonal like x-y axis
- maximum unknowns in 3d is 3
- max unknowns in 2d is 2
- use trig to solve direction
- moments | torque → cause rotation
- M_x, M_y, M_z equal 0
- 6 equilibrium equation ~ 6 unknown ← 30 mins
- suggest problems ~ Hibbles Textbook
- MP102 ~ tutorials thursday
- assingment → 1 min before tutorial starts
  → * beside 2 of them to submit at the end of the tutorial

Lecture #2

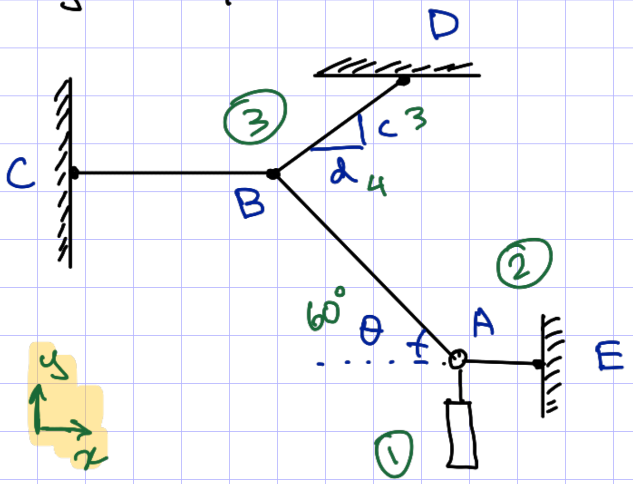
Sep 12, 2023

Eg 1: A pipe is supported at A by 5 cords. Determine the force in each cord for equilibrium

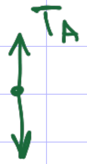
$m = 30$ kg
$c = 3$
$d = 4$
$\theta = 60$
$g = 9.8\, m/s^2$

Steps

1. draw FBD $\left\langle \begin{matrix}\text{known}\\ \text{unknown}\end{matrix}\right.$
2. resolve all forces into x and y
3. apply equilibrium equations
4. solve for unknowns



① FBD at weight

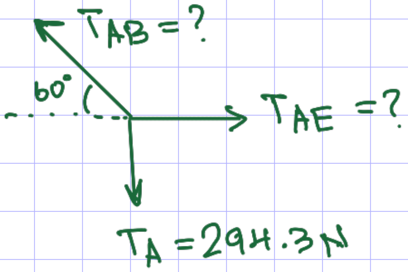


$\omega = mg$
$= 30 \times 9.8$
$= 294.3N \downarrow$

$+\uparrow \sum F_y = 0$

$T_A - \omega = 0$
$T_A = 294.3N$
$\approx 294 N$

② FBD at Joint A



$T_{ABx} = -T_{AB}\cos 60°$
$T_{ABy} = T_{AB}\sin 60°$

$+\uparrow \sum F_y = 0$

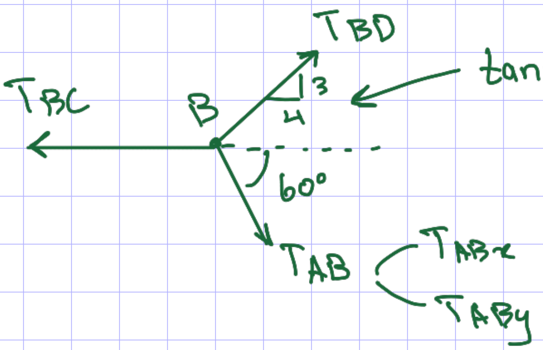
$-294.3 + T_{AB}\sin 60° = 0$
$T_{AB} = 339.83\ N$
$\approx 340\ N$

$\overset{+}{\rightarrow} \sum F_x = 0$

$+T_{AE} - T_{AB}\cos 60° = 0$
$T_{AE} = T_{AB}\cos 60°$
$= 339.83\cos 60°$
$T_{AE} = 169.9\ N$

③ FBD at Joint B

resolve T_{BD} in x, y



$\tan^{-1}\frac{3}{4} = 36.87°$

$$T_{BDx} = T_{BD}\cos 36.87°$$

$$T_{BDy} = T_{BD}\sin 36.87°$$

$$\overset{+}{\uparrow} \sum F_y = 0$$

$$+T_{BD}\sin 36.87° - T_{AB}\sin 60 = 0$$

$$T_{BD} = 490\ N$$

$$\overset{+}{\rightarrow} \sum F_x = 0$$

$$-T_{BC} + T_{BDx} + T_{ABx} = 0$$

$$-T_{BC} + T_{BD}\cos 36.87 + T_{AB}\sin 60° = 0$$

$$T_{BC} = 490\cos 36.87° + 339.83\sin 60°$$

$$= 686.3\ N$$

Lecture #2

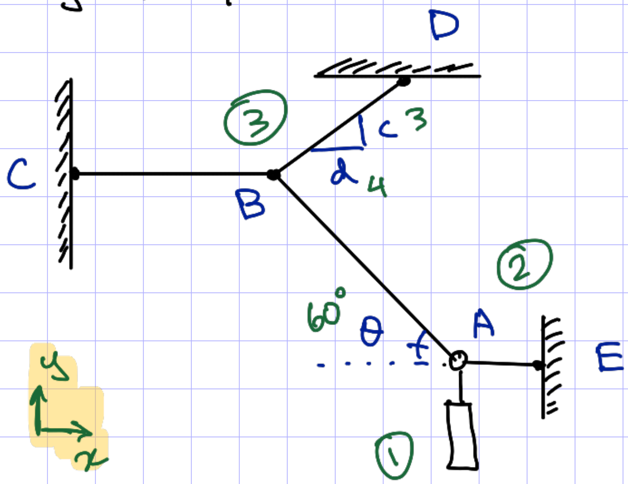
Sep 12, 2023

Eg 1: A pipe is supported at A by 5 cords. Determine the force in each cord for equilibrium

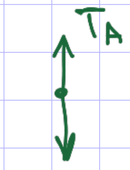
$m = 30$ kg
$c = 3$
$d = 4$
$\theta = 60$
$g = 9.8\ m/s^2$

Steps

1. draw FBD < known, unknown
2. resolve all forces into x and y
3. apply equilibrium equations
4. solve for unknowns



① FBD at weight

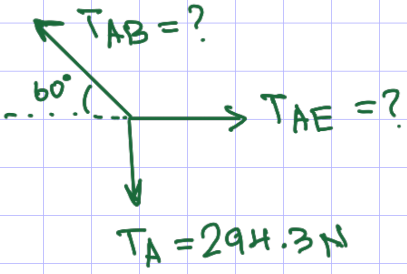


$w = mg$
$= 30 \times 9.8$
$= 294.3N \downarrow$

$+\uparrow \sum F_y = 0$

$T_A - w = 0$
$T_A = 294.3 N$
$\approx 294 N$

② FBD at Joint A



$T_{ABx} = - T_{AB} \cos 60°$
$T_{ABy} = T_{AB} \sin 60°$

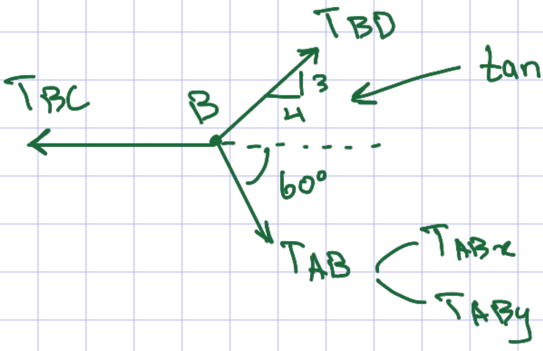
$+\uparrow \sum F_y = 0$

$-294.3 + T_{AB} \sin 60° = 0$
$T_{AB} = 339.83\ N$
$\approx 340\ N$

$\overset{+}{\rightarrow} \sum F_x = 0$

$+T_{AE} - T_{AB} \cos 60° = 0$
$T_{AE} = T_{AB} \cos 60°$
$= 339.83 \cos 60°$
$T_{AE} = 169.9\ N$

③ FBD at Joint B



$\tan\frac{3}{4} = 36.87^\circ$

resolve TBD in x,y

$T_{BDx} = T_{BD}\cos 36.87^\circ$

$T_{BDy} = T_{BD}\sin 36.87^\circ$

$\overset{+}{\uparrow}\sum F_y = 0$

$+T_{BD}\sin 36.87^\circ - T_{AB}\sin 60 = 0$

$T_{BD} = 490\ N$

$\overset{+}{\rightarrow}\sum F_x = 0$

$-T_{BC} + T_{BDx} + T_{ABx} = 0$

$-T_{BC} + T_{BD}\cos 36.87 + T_{AB}\sin 60^\circ = 0$

$T_{BC} = 490\cos 36.87^\circ + 339.83\sin 60^\circ$

$= 686.3\ N$

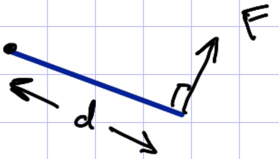
Lecture #3

Sep 14, 2023

Unit 2 : Rigid Body, Moments

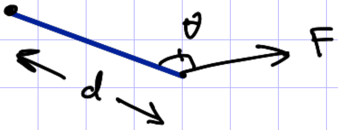
Moment or Torque : tendency of a force to cause a body to rotate about a point or axis

Scenario 1 :



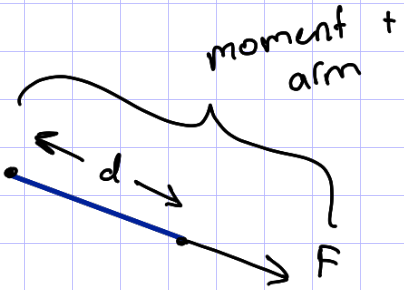
- max moment when $d \perp F$

Scenario 2:



- F appied where $\theta \neq 90°$
- difficult to turn or rotate body

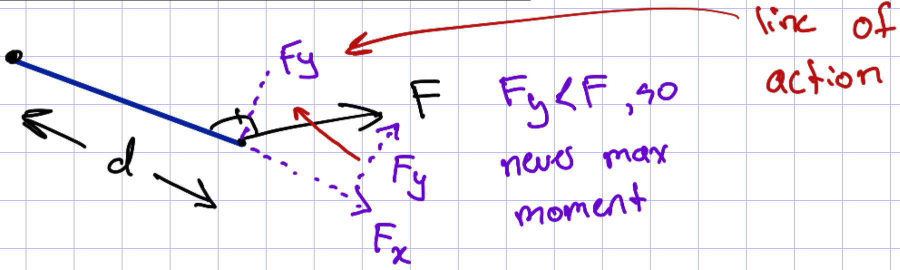
Scenario 3:



- no d (no moment arm)
- F ∥ object

  $M=0$ (minimum moment)

Resolve into Components



$F_y < F$, so never max moment

Magnitude

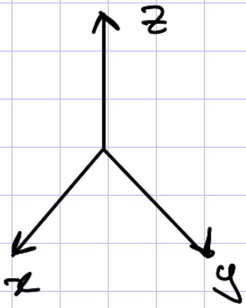
$\boxed{M_O = Fd}$ (N·m) ← units of moments

$d = \perp$ distance from "O" to its line of action

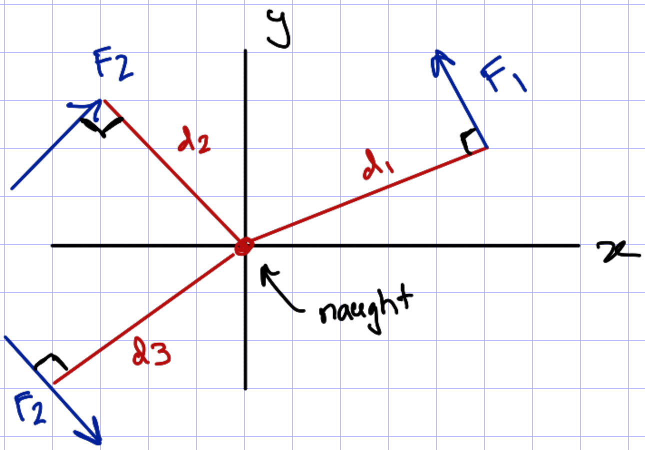
Direction

in 2D → CW (clockwise) or CCW (counterclockwise)

in 3D → Right hand rule



Resultant

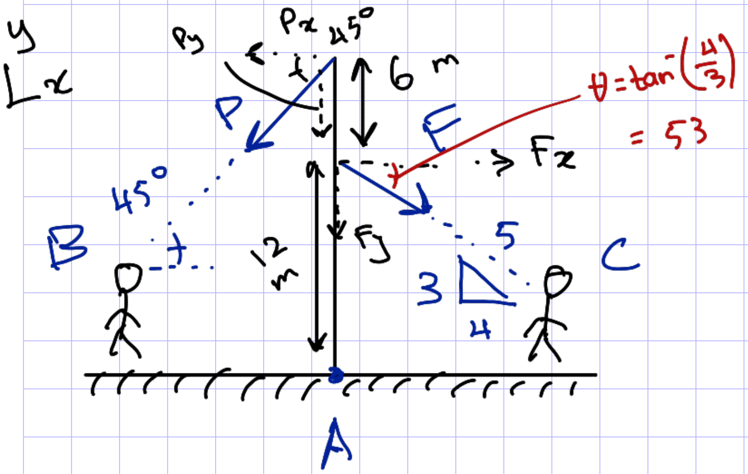


$+\circlearrowleft \; M_{R_O} = \sum F \times d$

$M_{R_O} = F_1 d_1 - F_2 d_2 + F_3 d_3$

Eg: 2 people exert forces F = 80 N, P = 50 N on ropes. Determine the moment of each force about A.

a) Which way will the people rotate (cw or ccw)?



Step 1: FBD, set axis
2: Resolve P, F in x, y
3: calculate moment M (f×d) at a A of P, F
4: indicate magnitude and direction

Resolve P

$P_x = -P\cos 45°$

$P_y = -P\sin 45°$

Resolve F

$F_x = +F\cos 36.87°$

$F_y = -F\sin 36.87°$

$$+\circlearrowright \; M_A^P = P_x d + \overset{0}{\cancel{P_y d}}$$
$$= P_x(18)$$
$$= -P\cos 45° \times 18$$
$$= -50\cos 45° (18)$$
$$= -636.4\,N \;\; N\cdot m \; \circlearrowleft$$

$$+\circlearrowright \; M_A^F = F_x d + \overset{0}{\cancel{F_y d}}$$
$$= F_x(12)$$
$$= +F\cos 36.87° (12)$$
$$= +80\cos 36.87° (12)$$
$$= 768 \; N\cdot m \; \circlearrowright$$

a) Pole will rotate clockwise

b) What force does person C have to exert for there to be no moment (no rotation) at A?

$\boxed{+\curvearrowright \ \sum M_A = 0}$

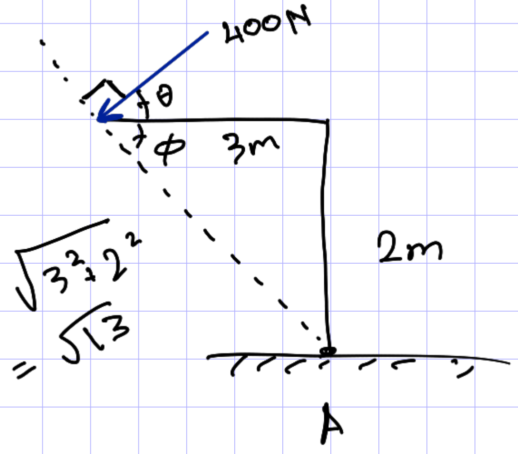
General $-P_x d_p + F_x d_y = 0$

unknown F

$50\cos 45(18) = F\cos 36.87(12)$

$$F = \frac{50\cos 48(18)}{\cos 36.87(12)}$$

$F = 66.3\ N$



Find θ for a) max and b) min moment about A.

$\theta + \phi = 90°$

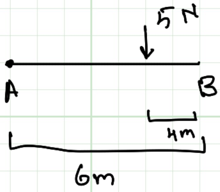
$\phi = \tan^{-1}\left(\frac{2}{3}\right)$

$\theta = 90° - \tan^{-1}\left(\frac{2}{3}\right) = 56.3°$

$\beta = \theta + 90°$

$= 146°$

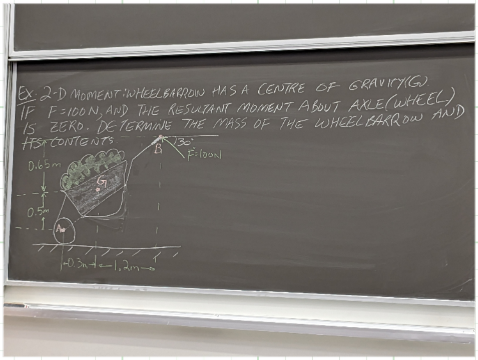
Week 2: Lecture #1 | Sep 18, 2023 | 2-D Moments



$$+\circlearrowright\ M_A = Fd$$
$$= 5 \times 6 = \boxed{30\ \mathrm{N \cdot m}\ \circlearrowright}$$

$$+\circlearrowright\ M_B = Fd$$
$$= -(5 \times 4) = \boxed{-20\ \mathrm{N \cdot m}\ \circlearrowright}$$
$$= \boxed{+20\ \mathrm{N \cdot m}\ \circlearrowleft}$$

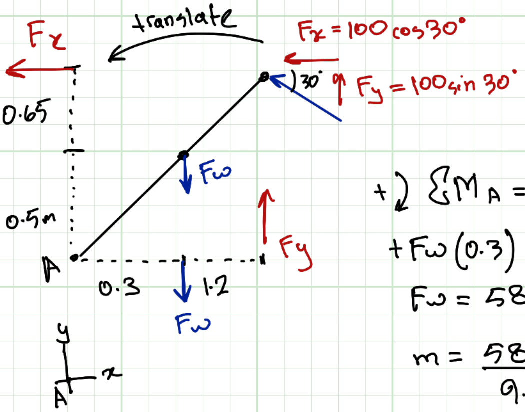
Eg 1 :



Ex. 2-D MOMENT: WHEELBARROW HAS A CENTRE OF GRAVITY (G). IF F=100N, AND THE RESULTANT MOMENT ABOUT AXLE (WHEEL) IS ZERO. DETERMINE THE MASS OF THE WHEELBARROW AND ITS CONTENTS.

Steps

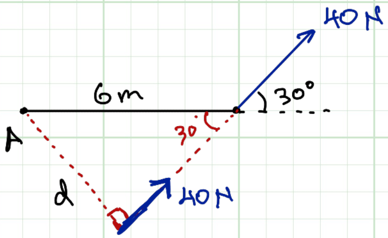
1. Draw FBD — known / unknown
2. set x, y axes
3. Resolve Forces in x, y
4. Calculate $M_A = 0$

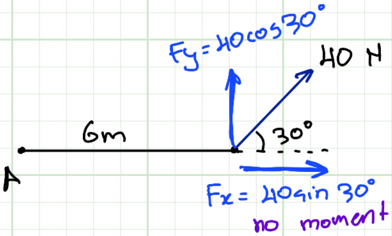


$F_\omega =$

$$+\circlearrowright\ \Sigma M_A = 0$$
$$+F_\omega(0.3) - F\cos 30(1.15) - F\sin 30(1.5) = 0$$
$$F_\omega = 581.98\ \mathrm{N} = mg$$
$$m = \frac{581.98}{9.81} = 59.3\ \mathrm{kg}$$
$$\therefore m = 59.3\ \mathrm{kg}$$

2 Ways to Calculate Moments





$d = 6\sin 30°$

$M_A = F \times d$

$= 4\,(6\sin 30°)$

$M_A = 120\ N\cdot m$ ↺

$M_A = F \times d$

$= \cancel{F_x d}^{\,0} + F_y d$

$= F_y d$

$= 40\sin 30° \times 6m$

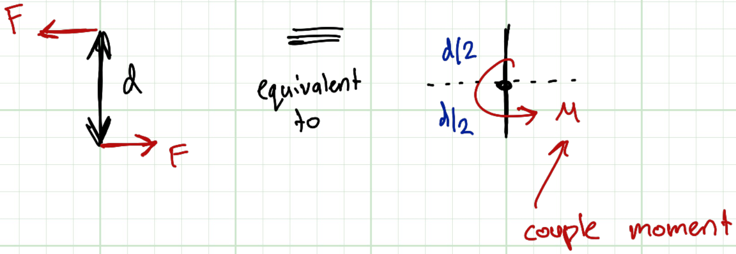
$M_A = 120\ N\cdot m$ ↺

Couple : moments generated by a couple force

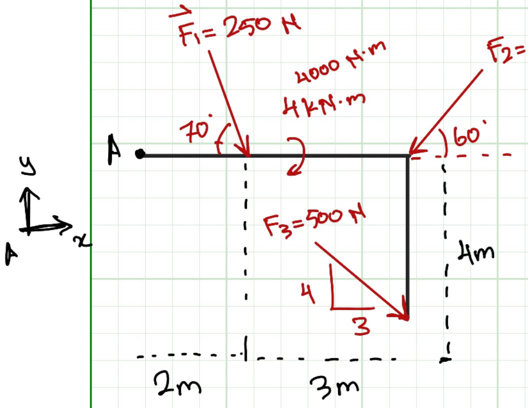
- 2 parallel forces that are equal in magnitude and opposite in direction

+

- separated by perpendicular distance



Eg 2: What is the rotational effect about Point A from all 3 forces and the couple?



Steps:

1. Draw FBD + axes
2. Resolve F
3. $M_A \longrightarrow f(F_1, F_2, F_3, M)$

$M_A =$

① Resolve F_1, F_2, F_3

$\vec{F_1} = 250$ N

$F_{1x} = 250\cos 70° = 85.51$

$F_{1y} = -250\sin 70° = -234.92$

$F_2 = 300$ N

$F_{2x} = -300\cos 60° = -150.0$

$F_{2y} = -300\sin 60° = -259.81$

$F_3 = 500$ N

$F_{3x} = 500\cos 53 = 300$

$F_{3y} = -500\sin 53° = -399.84$

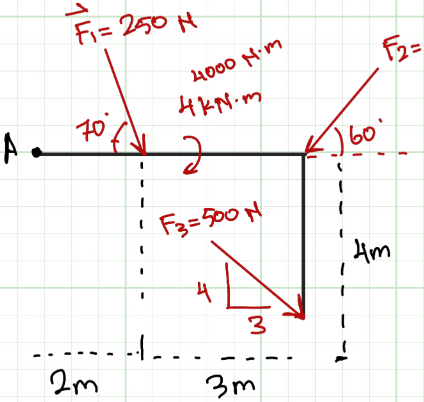
$$+\circlearrowright M_A = +F_{1y}(2) + F_{2y}(5) - F_{3x}(4) + F_{3y}(5) + 4000$$

$$= 6568 \text{ N·m} \approx 6.57 \text{ kN·m } +\circlearrowright$$

Week 2: Lecture #2

From Last Class:

b) Find resultant force and moment at A (+direction) Draw equivalent (resultant system)



from last class:

$$+\circlearrowright M_A = 6.57 \text{ kN}\cdot\text{m} \circlearrowright$$

$$\vec{F}_R = F_1 + F_2 + F_3$$

x-comp: $F_{1x} + F_{2x} + F_{3x}$

$$\Sigma F_x = 235.51 \text{ N} \rightarrow$$

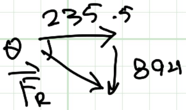
y-comp: $F_{1y} + F_{2y} + F_{3y}$

$$\Sigma F_y = 894.57 \text{ N} \downarrow$$

$$|\vec{F}_R| = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

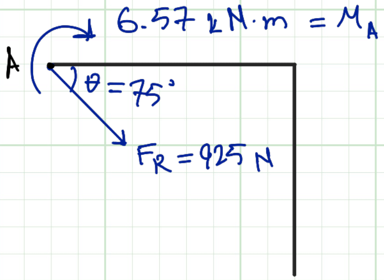
$$= \sqrt{235.51^2 + 894.57^2}$$

$$= 925 \text{ N}$$

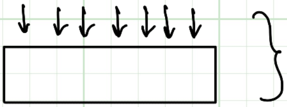


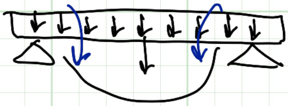
$$\theta = 75.25 \approx 75°$$

Draw Resultant System

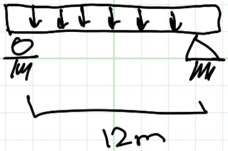


Uniformly Distributed Load (UDL)

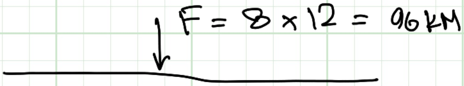




8 kN/m = ω (N/m)



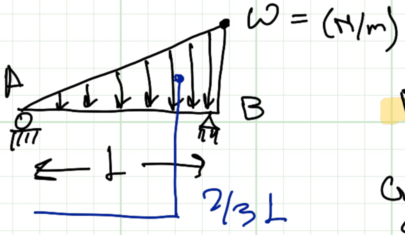
rectangularly distributed load



Force = ωL

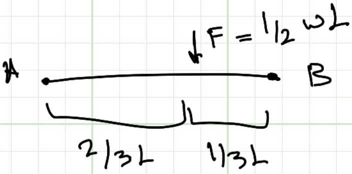
Geometric Centroid = L/2

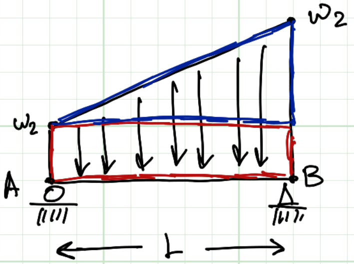
→ rectangular distributed load



$F = \frac{1}{2}\omega L$

Geometric Centroid $= x = \frac{2}{3}L$ from A



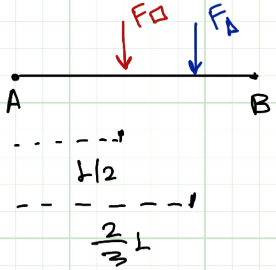


$F_{\square}$ ↓

$F_{\square} = \omega_1 L$ at $L/2$

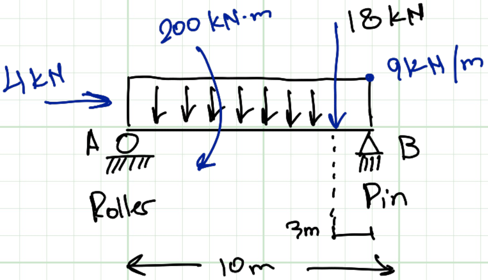
F_{Δ} ↓

$F_{\Delta} = \frac{1}{2} L(\omega_2 - \omega_1)$ at $\frac{2}{3} L$



Hibbeler Ch.5 -
table 5.1 – 5.2 "supports"

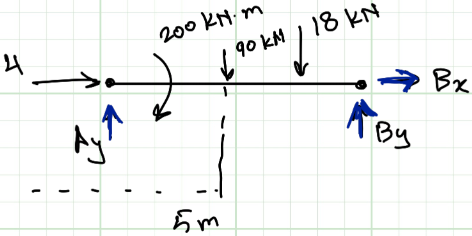
Eg: Find support reaction (for equilibrium)



$F_{\square} = 9 \times 10 = 90$ kN act at $\frac{L}{2}$

3 uknown: $\overset{+}{\curvearrowright} \Sigma F_x = 0$ $\quad \overset{+}{\rightarrow} \Sigma M = 0$

$+\uparrow \Sigma F_y = 0$



$\overset{+}{\rightarrow} \sum F_x = 0 \qquad +4 + B_x = 0 \qquad \underset{\longrightarrow}{B_x = -4\text{ kN}} \qquad \underset{\longleftarrow}{B_x = 4\text{ kN}}$

$+\uparrow \sum F_y = 0 \qquad A_y - 90 - 18 + B_y = 0$

$A_y + B_y = 108$ return

$+\circlearrowleft M_A = 0 \qquad +200 + 90(5) + 18(7) - B_y(10) = 0$

$B_y = 77.6\text{ kN} \uparrow$

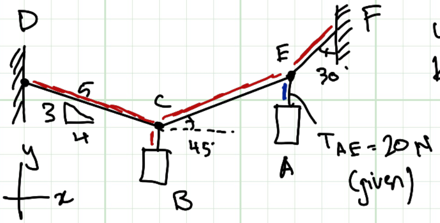
$A_y = 108 - 77.6$

$= 30.4\text{ kN} \uparrow = A_y$

Week 2: lec 3

Eg: 2-D System in Equilibrium

Determine the weight "B" for the system to be in equilibrium



Uknowns: T_{CD}, T_{EF}, T_{Ec}, W_B = ?

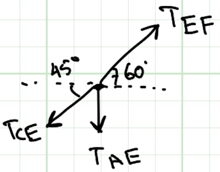
Knowns: $T_{AE} = 20$ N

Steps: FBD axis inspection
↳ where to start?

Joint C

Joint E

Joint E



$T_{EF_x} = T_{EF} \cos 60°$

$T_{EF_y} = T_{EF} \sin 60°$

$T_{CE_x} = T_{CE} \cos 45°$

$T_{CE_y} = T_{CE} \sin 45°$

$T_{AE_y} = 20$ N

$T_{EF} = 54.6$ N

$T_{EC} = 38.6$ N

P

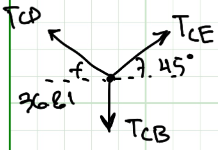
$0 = T_{EF} \cos 60 - T_{CE} \cos 45°$

$20 = T_{EF} \sin 60° - T_{CE} \sin 45°$

$\overset{+}{\rightarrow} \sum F_x = T_{EF} \cos 60 - T_{CE} \cos 45°$

$+\uparrow \sum F_y = T_{EF} \sin 60 - T_{CE} \sin 45° - 20$

Joint C

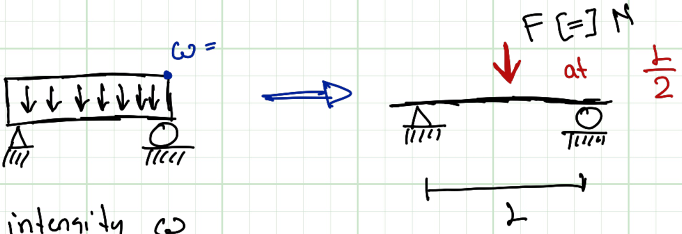


$\overset{+}{\rightarrow} \sum F_x = T_{CE} \cos 45° - T_{CD} \cos 36.81$ P

$+\uparrow \sum F_y = T_{CE} \sin 45° + T_{CD} \sin 36.81 - T_{CB}$

$T_{CD} = 34.1$ N $T_{CB} = 47.8$ N

Distributed Load



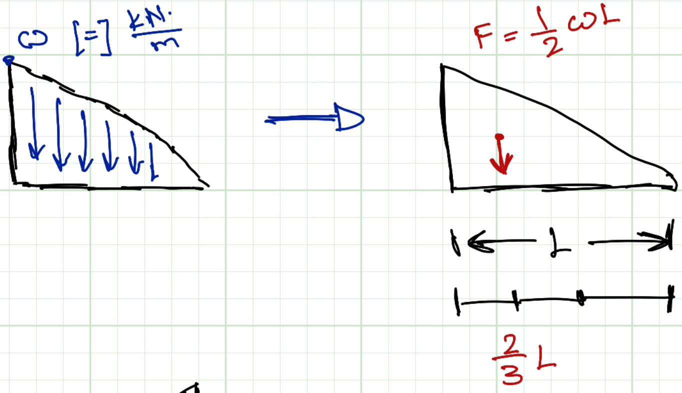
• intensity ω

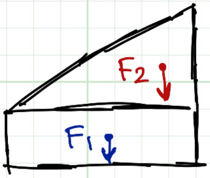
units $\left(\frac{N}{m}\right)$

$= \omega \times L$

$= \frac{N}{m} \times m = N$

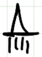
$F = \frac{1}{2} \omega L$

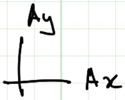


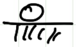


$F_2 = \frac{1}{2} L (\omega_2 - \omega_1)$ at $\frac{2}{3} L$

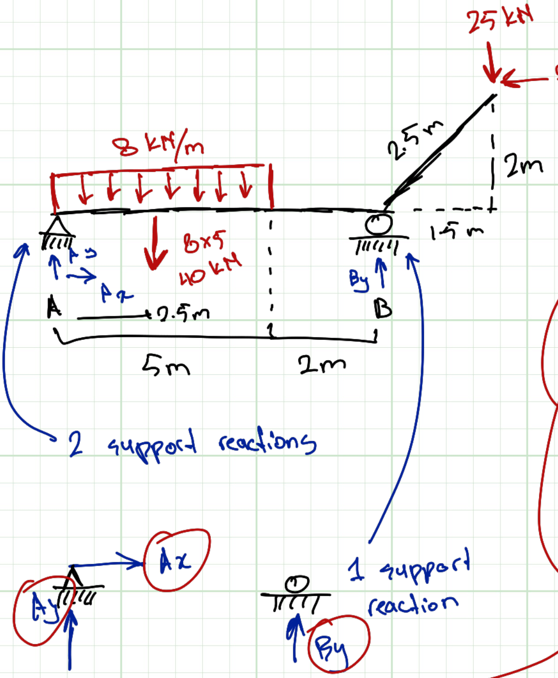
$F_1 = \omega L$ at $\frac{L}{2}$


"pin" ~ 2 restraints




"rolles" ~ 1 restraints

Eg 2: Find reaction supports



don't start with this bc. Ay and By are unknown

$\uparrow^+ \sum F_y = 0$

$\xrightarrow{+} \sum F_x = 0$

$+\circlearrowright M_o = 0$

$\sum F_x = 0$

$+ A_x - 30 = 0$

$A_x = 30$ kN →

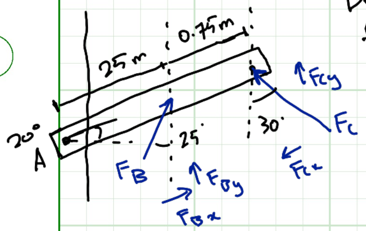
$$40(2.5m) - B_y(7m) - 30(2m) + 25(5+2+1.5) = 0$$

$B_y = +36.1$ kN ↑

$\uparrow^+ \sum F_y = 0$

$+A_y - 40 + B_y - 25 = 0$

$A_y = 28.9$ kN $\uparrow^*$

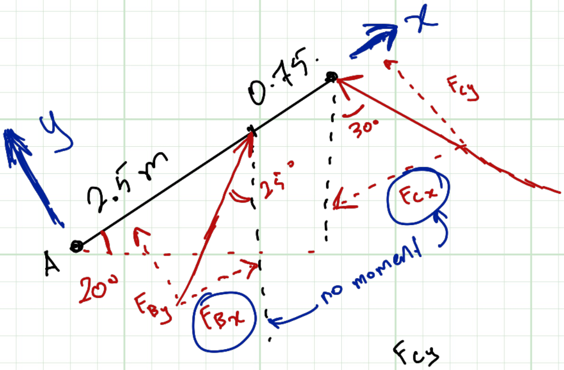


Determine the moment of each force about the bolt located at A.

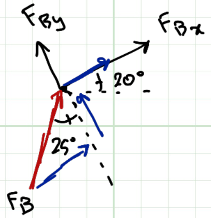
$F_B = 40$ N , $F_C = 50$ N

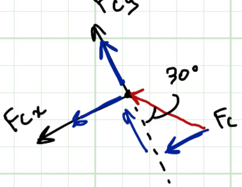
Step 1: Draw FBD

2: Set x-y-axes



$$\therefore M_A^B = 90.6 \text{ N·m} \circlearrowleft$$
$$M_A^C = 140.7 \text{ N·m} \circlearrowleft$$





$F_{Bx} = F_B \sin 25°$

$F_{By} = F_B \cos 25°$

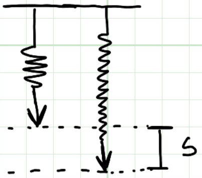
$F_{Cx} = F_C \sin 30°$

$F_{Cy} = F_C \cos 30°$

$$M_A^B = (F_B \cos 25°)(2.5)$$
$$= (40 \cos 25°)(2.5)$$
$$= 90.63 \text{ N·m} \circlearrowleft$$

$$M_A^C = (F_C \cos 30°)(2.5 + .75)$$
$$= (50 \cos 30°)(2.5 + 0.75)$$
$$= 140.7 \text{ N·m} \circlearrowleft$$

1) Springs : elastic and linear



$F_{spring} \propto S$

displacement

force in spring

proportional to

$$\boxed{F = kS}$$

spring constant

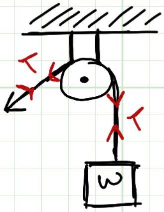
elongation or compression

F: in Newtons (N)

k: in Newtons/metre $\left(\frac{N}{m}\right)$ ~ stiffness or elasticity

S: in metres (m)

2) Cables and Pulleys



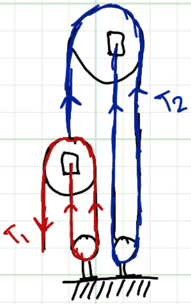
- used to redirect forces
- use less force to lift objects

1) the force in a single cable going around a pulley is constant

at any angle

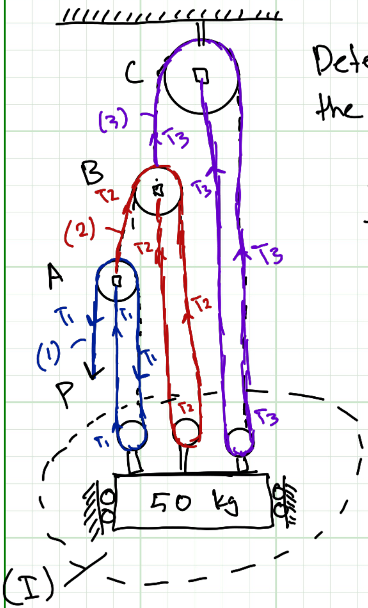
2) Force in cable is always tension

- assume pulley is frictionless



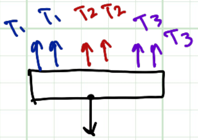
Week 3: Lecture 2

Fg 1



Determine the force P required to hold the 50 kg mass in equilibrium

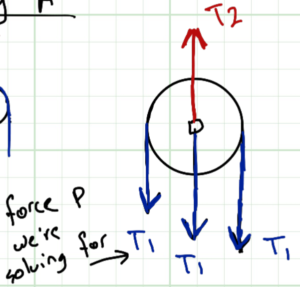
FBD I



50×9.81 = 490.5 N = ω

$$\uparrow^{+} \Sigma F_y = 0 = \boxed{2T_1 + 2T_2 + 2T_3 = \omega} \quad A$$

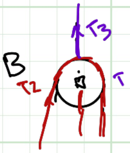
Pulley A

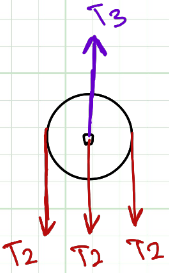


$$\uparrow^{+} \Sigma F_y = 0 = T_2 - 3T_1$$

$$\boxed{3T_1 = T_2} \quad B$$

Pulley B





$$\uparrow^{+} \Sigma F_y = 0 = T_3 - 3T_2$$

$$\boxed{3T_2 = T_3} \quad C$$

Substitut Ⓑ and Ⓒ into Ⓐ

$\boxed{2T_1+2T_2+2T_3=\omega}$ $\boxed{3T_1=T_2}$ $\boxed{3T_2=T_3}$

$$2T_1+2(3T_1)+2(3T2)=\omega$$

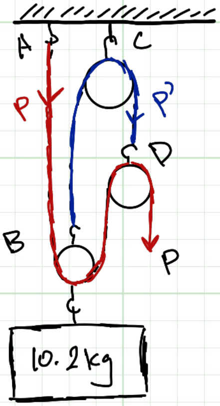
$$2T_1+6T_1+6(3T_1)=\omega$$

$$2T_1+6T_1+18T_1=\omega$$

$$26T_1=\omega$$

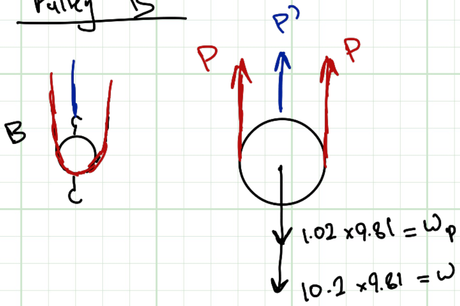
$$T_1=P=\frac{\omega}{26}=\frac{490.5}{26}=\boxed{18.87\ N}$$

Eg 2



Determine the P needed to support 10.2 kg weight. Each pulley is 1.02 kg

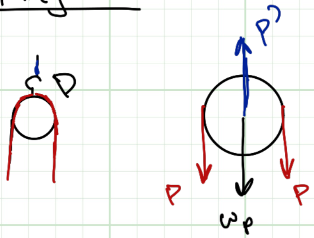
Pulley B



$$\overset{+}{\uparrow}\Sigma F_y=0=P'+2P-\omega_p-\omega$$

$$\boxed{P'+2P=\omega_p+\omega}$$

Pulley D



$$\uparrow^{+}\Sigma F_y = 0 = P' - 2P - \omega_p$$

$$\boxed{P' = 2P + \omega_p}$$

Plug and Solve

$$\boxed{P' + 2P = \omega_p + \omega} \qquad \boxed{P' = 2P + \omega_p}$$

$1.02 \times 9.81 = \omega_p$

$10.2 \times 9.81 = \omega$

$$P' + 2P = \omega_p + \omega$$

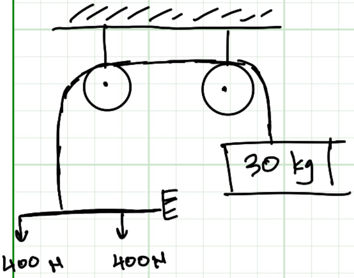
$$2P + \cancel{\omega}_p + 2p = \cancel{\omega}_p + \omega$$

$$4p = \omega$$

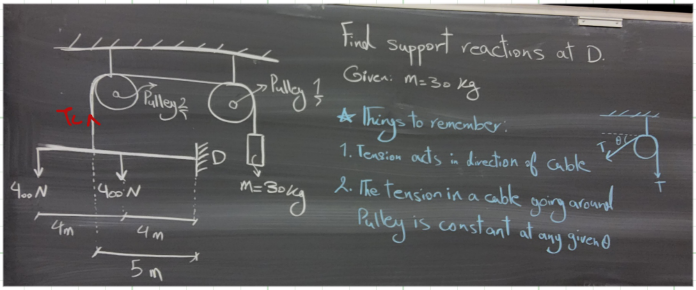
$$p = \frac{\omega}{4}$$

$$= \frac{(10.2 \times 9.81)}{4} = 25.02\ N$$

Eg3



Week 3: Lecture 3



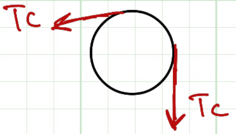
Find support reactions at D.

Given: m=30 kg

★ Things to remember:

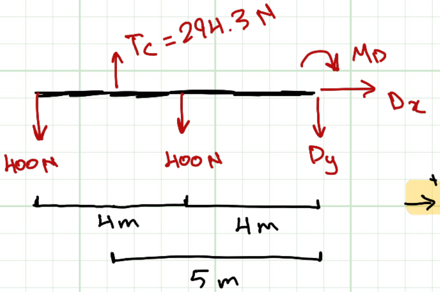
1. Tension acts in direction of cable
2. The tension in a cable going around Pulley is constant at any given θ

FBD of Pulley 1



$$T_c = mg = (30\,kg)(9.8\,m/s^2) = 294.3\ N$$

FBD of Beam



- fixed support : 3 reactions
- beam can't go up/down : F_{Dy} ↓
- beam can't go right/left : F_{Dx} →
- beam can't rotate : M_D ↺

$$\overset{+}{\rightarrow}\sum F_x = 0 = D_x$$

- to start with

$$+\uparrow \sum F_y = 0 = -400 - 400 - D_y + 294.3$$

$$D_y = -400 - 400 + 294.3$$

$$= -505.7\ N \uparrow = 505.7\ N \downarrow$$

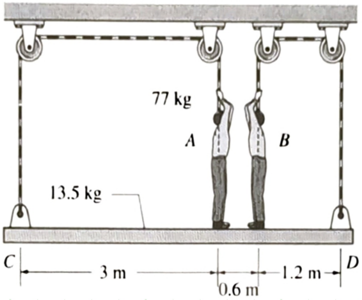
$$+\circlearrowleft \sum M_D = 0 = -400(8) - 400(4) + 294.3(5) + M_D$$

$$M_D = 400(8) + 400(4) - 294.3(5)$$

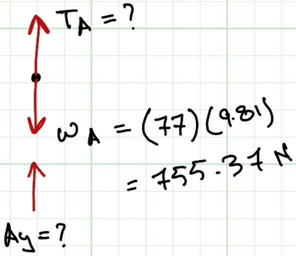
$$= 3328.5\ N\cdot m \circlearrowleft$$

$$\boxed{\therefore D_x = 0N,\ D_y = 505.7\ N \uparrow,\ M_D = 3328.5\ N\cdot m \circlearrowleft}$$

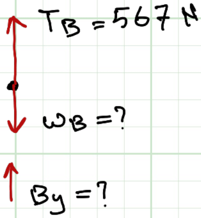
The window washers *A* and *B* support themselves and the horizontal, uniform platform *CD* (i.e. the weight of the platform acts at its center) by pulling down on the two ropes. The pulling force on the rope by the washer *B* is **567 N**. The mass of washer A is **77 kg**. Determine the tension in the rope connected at *C*, the mass of the washer *B*, and the force that each man exerts on the platform.



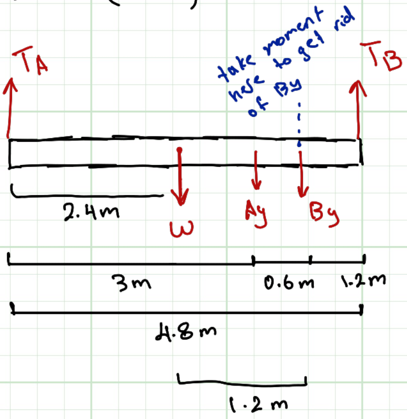
FBD 1 (man A)



FBD 2 (man B)



FBD 3 (beam)



$W = 13.5(9.8)$

FBD 1

$\uparrow^{+}\sum F_y = 0 = T_A + A_y - W_A$

$W_A = T_A + A_y$

I $\boxed{T_A + A_y = 755.37 \text{ N}}$

FBD 2

$\uparrow^{+}\sum F_y = 0 = T_B - W_B + B_y$

II $\boxed{B_y - W_B = -567}$

FBD 3

$+\circlearrowleft \sum M_B = 0 = T_A(3.6) - \omega_B(1.2) + A_y(0.6) - T_B(1.2)$

III $\boxed{3.6(T_A) - 0.6(A_y) = 839.28}$

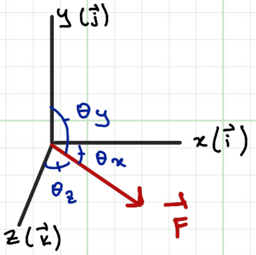
$+\uparrow \sum F_y = 0 = T_A - \omega - A_y - B_y + T_B$

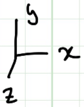
IV $\boxed{T_A - A_y - B_y = -567 + 132.43 = -434.6}$

$$\boxed{\begin{aligned} T_A &= 308 \text{ N} \\ A_y &= 448 \text{ N} \\ B_y &= 295 \text{ N} \\ m_B &= 87.9 \text{ kg} \end{aligned}}$$

Week 4: lecture #1

Statics in 3D





- Resolving $\vec{F}$ in 3-D

↳ if given θ_x, θ_y, θ_z which are angles from $\vec{F}$ to x, y, z axes

$$\vec{F} = |F|\cos\theta_x \vec{i} + |F|\cos\theta_y \vec{j} + |F|\cos\theta_z \vec{k}$$

- given angles

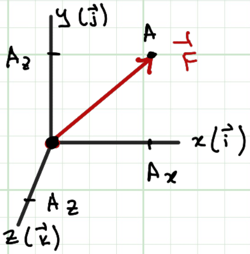
↑ i component ↑ j component ↖ k component

Resultant $|F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

↓ $|F|\cos\theta_x$ ↓ $|F|\cos\theta_y$ ↘ $|F|\cos\theta_z$

Case: Not Given θ_x, θ_y, θ_z

↳ if given position vector
(coordinates of head and tail of force vector)



$O = (0,0,0)$

$A = (a_x, a_y, a_z)$

Position Vector

$$\vec{r}_{OA} = (r_A - r_o)_x \vec{i} + (r_A - r_o)_y \vec{j} + (r_A - r_o)_z \vec{k}$$

$$\vec{r}_{OA} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$

Unit Vector

$$\vec{u}_{AO} = \frac{(r_{oa})_x \vec{i}}{|r_{oa}|} + \frac{(r_{oa})_y \vec{j}}{|r_{oa}|} + \frac{(r_{oa})_z \vec{k}}{|r_{oa}|}$$

check $\vec{\mu}_{ax} = 1 = \sqrt{\_^2 + \_^2 + \_^2}$

$$\vec{F} = |F| \vec{u}_{oa} = |F| u_{oax} \vec{i} + |F| u_{oay} \vec{j} + |F| u_{aoz} \vec{k}$$

$$= |F| \frac{ax}{|r_{oa}|} \vec{i} + |F| \frac{ay}{|r_{oa}|} \vec{j} + |F| \frac{az}{|r_{oa}|} \vec{k}$$

→ Eg 1: Resolve $F = 700$ N into x, y, z → $\vec{i}, \vec{j}, \vec{k}$

Given: $A = (0,0,0)$, $B = (6,3,4) \sim (x,y,z)$

$$\boxed{|\vec{F}| = |F| \vec{u}_{AB}}$$

↑

700

$$|r_{AB}| = \sqrt{(R_B - R_A)_x^2 + (R_B - R_A)_y^2 + (R_B - R_A)_z^2}$$

$$= \sqrt{6^2 + 3^2 + 4^2} = \sqrt{61}$$

$$\vec{u}_{AB} = \frac{(r_B - r_A)_x}{|r_{AB}|} \vec{i} + \frac{(r_B - r_A)_y}{|r_{AB}|} \vec{j} + \frac{(r_B - r_A)_z}{|r_{AB}|} \vec{k}$$

$$\vec{u}_{AB} = \frac{6}{\sqrt{61}} \vec{i} + \frac{3}{\sqrt{61}} \vec{j} + \frac{4}{\sqrt{61}} \vec{k}$$

$$\vec{u}_{AB} = 700\left(\frac{6}{\sqrt{61}}\right)\vec{i} + 700\left(\frac{3}{\sqrt{61}}\right)\vec{j} + 700\left(\frac{4}{\sqrt{61}}\right)\vec{k}$$

$$\vec{u}_{AB} = 537.8\vec{i} + 268.9\vec{j} + 358.5\vec{k}$$

check that $|\vec{u}_{AB}| \equiv 700$

$$\sqrt{537.8^2 + 268.9^2 + 358.5^2} \approx 700$$

→ Eg 2: 3D Equilibrium of Forces

weight is supported by 3 cables, what is w that the cables can carry?

Given: $|T_{AD}| = 4620$ N

$r_A = (0,0,0)$

$r_B = (700,1125,0)$

$r_C = (0,1125,-600)$

$r_D = (-650,1125,450)$

1) draw FBD

2) resolve

↳ T_{AD}

↳ T_{AB}

↳ T_{AC}

↳ w

3) solve w

↳ apply eq. equat.

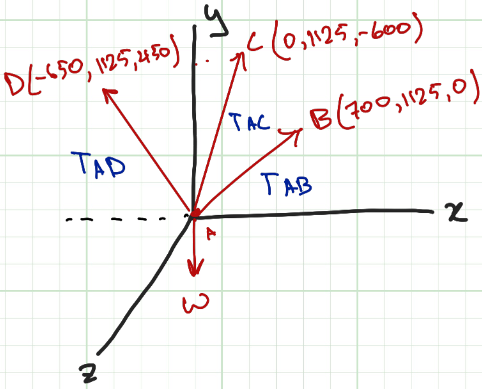
Unkown

$T_{AB} = ?$

$T_{AC} = ?$

$w = ?$

$\vec{F} = |F|\vec{u}$



→ $T_{AD} : |T_{AD}| \vec{u}_{AD}$

$$\vec{u}_{AD} = \frac{\vec{r}_{AD}^{\,*}}{|r_{AD}|^{*}} = \frac{-650}{1375}\vec{i} + \frac{1125}{11375}\vec{j} + \frac{450}{1375}\vec{k}$$

$$* \quad \vec{r}_{AD} = (r_D - r_A)_x \vec{i} + (r_D - r_A)_y \vec{j} + (r_D - r_A)_z \vec{k}$$

$$= -650\vec{i} + 1125\vec{j} + 450\vec{k}$$

$$* \quad |\vec{r}_{AD}| = \sqrt{650^2 + 1125^2 + 450^2} = 1375$$

$$T_{AD} = |T_{AD}| \vec{u}_{AD}$$

$$= -2185\vec{i} + 3780\vec{j} + 1512\vec{k}$$

→ $T_{AB} = |T_{AB}| \vec{u}_{AB}$ ← don't have this

$$r_{AB} = \sqrt{700^2 + 1125^2 + 0^2}$$

$$= 1325$$

$$\vec{u}_{AB} = \frac{700}{1325}\vec{i} + \frac{1125}{1325}\vec{j} + 0\vec{k}$$

$$T_{AB} = 0.5283 T_{AB}\vec{i} + 0.849 T_{AB}\vec{j} + 0 T_{AB}\vec{k}$$

→ $T_{AC} = |T_{AC}| \vec{u}_{AC}$

$$\vec{u}_{AC} = \frac{F_{AC}}{|r_{AC}|} = \frac{0\vec{i} + 1125\vec{j} - 600\vec{k}}{\sqrt{0^2 + 1125^2 + (-600)^2}} = 0.88235\vec{j} - 0.470\vec{k}$$

$$\vec{T}_{ac} = 0\vec{i} + 0.88235 T_{ac}\vec{j} - 0.470 T_{ac}\vec{k}$$

→ $\omega = -\omega\vec{j}$

	x	y	z
T_{AB}	$+0.5283\underline{T_{AB}}$	$+0.849\underline{T_{AB}}$	0
T_{AC}	0	$+0.882\underline{T_{AC}}$	$-0.471\underline{T_{AC}}$
T_{AD}	-2185	$+3780$	1515
T_w	0	$-\underline{w}$	0
	$\Sigma F_x = 0$	$\Sigma F_y = 0$	$\Sigma F_z = 0$

$\Sigma F_x = 0 = 0.5283 T_{AB} - 2185$

$T_{AB} = 4136 N$

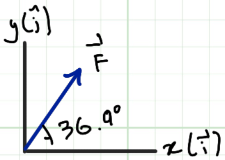
$\Sigma F_z = 0 = -0.471 T_{AC} + 1515$

$T_{AC} = 3210.2 N$

$\Sigma F_y =$

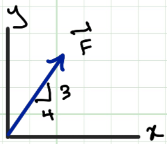
$w = 10123 N$

Week 4: Lecture #2



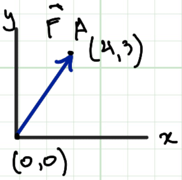
$F_x = F\cos 36.9^\circ$

$F_y = F\sin 36.9^\circ$



$F_x = F\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$

$F_y = F\sin\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$



$\vec{F} = |F|\,\vec{u}_{oa}$

$|r_{oa}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

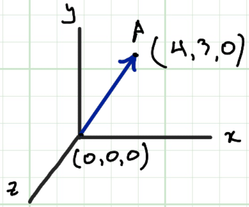
$\vec{u}_{oa} = \frac{\vec{r}_{oa}}{|r_{oa}|} = \frac{(4-0)\hat{i}}{5} + \frac{(3-0)\hat{j}}{5}$

$= \frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$

$\vec{F} = |F|\frac{4}{5}\hat{i} + |F|\frac{3}{5}\hat{j}$

$F_x = F\frac{4}{5}\hat{i}$

$F_y = F\frac{3}{5}\hat{j}$



$\vec{F} = |F|\,\vec{u}_{oa}$

$|r_{oa}| = \sqrt{4^2+3^2+0^2} = \sqrt{25} = 5$

$\vec{u}_{oa} = \frac{4}{5}\hat{i} + \frac{3}{5}\hat{j} + \frac{0}{5}\hat{k}$

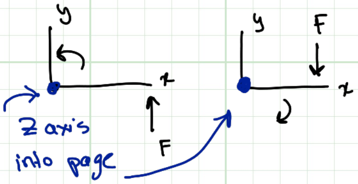
$\vec{F} = |F|\frac{4}{5}\hat{i} + |F|\frac{3}{5}\hat{j} + |F|\frac{0}{5}\hat{k}$

$F_x = F\frac{4}{5}\hat{i}$

$F_y = F\frac{3}{5}\hat{j}$

$F_z = F\frac{0}{5}\hat{k}$

3D Moments



moment about z axis

3D Cartesian Vector Formulation

$$\vec{C} = \vec{A} \times \vec{B}$$

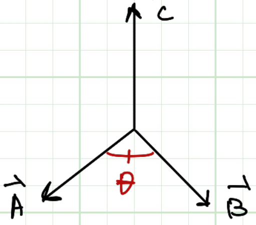
$$|C| = |A||B| \sin\theta$$

where θ is angle between the tails

$$0 < \theta < 180$$

$$\boxed{\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}}$$

same magnitude, but direction is different



$$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x\cancelto{0}{B_x(\hat{i} \times \hat{i})} + A_xB_y(\hat{i} \times \hat{j}) + A_xB_z(\hat{i} \times \hat{k})$$
$$+ A_yB_x(\hat{j} \times \hat{i}) + A_yB_y(\cancelto{0}{\hat{j} \times \hat{j}}) + A_yB_z(\hat{j} \times \hat{k})$$
$$+ A_zB_x(\hat{k} \times \hat{i}) + A_zB_y(\hat{k} \times \hat{j}) + A_zB_z(\cancelto{0}{\hat{k} \times \hat{k}})$$

$$= \underbrace{(A_yB_z - A_zB_c)}_{\hat{i}\text{ component}}\hat{i} - \underbrace{(A_xB_z - B_zB_x)}_{\hat{j}\text{ component}}\hat{j} + \underbrace{(A_xB_y - A_yB_x)}_{\hat{k}\text{ component}}\hat{k}$$

$\hat{i} \times \hat{i} = 0$ $\quad$ $\hat{i} \times \hat{j} = \hat{k}$ $\quad$ $\hat{i} \times \hat{k} = -\hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$ $\quad$ $\hat{j} \times \hat{j} = 0$ $\quad$ $\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad$ $\hat{k} \times \hat{j} = -\hat{i}$ $\quad$ $\hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{i} = 0$

$(1)(1)\sin 0 = 0$

$\hat{i} \times \hat{j} = 1\hat{k}$

$(1)(1)\sin 90 = 1$

Determinant Form

$$\vec{C} = \vec{A} \times \vec{B} = \left| \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array} \right| \begin{array}{l} \text{... unit vectors} \\ \text{... xyz comp. of A} \\ \text{... xyz comp. of B} \end{array}$$

$$\vec{M} = \underline{\vec{D}} \times \underline{\vec{F}}$$

element $\hat{i}$:

$$\left| \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array} \right| = (A_yB_z - A_zB_y)\,\hat{i}$$

element $\hat{j}$:

$$\left| \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array} \right| = -(A_xB_z - A_zB_x)\,\hat{j}$$

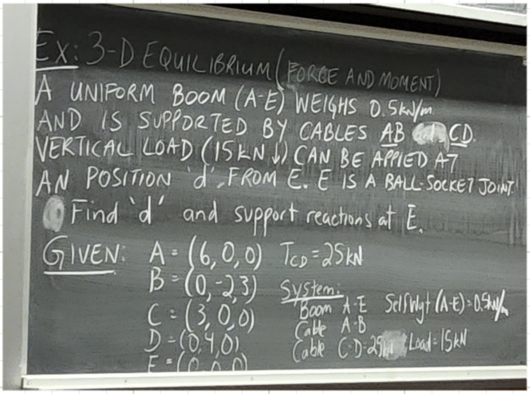
element $\hat{k}$:

$$\left| \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array} \right| = (A_xB_y - A_yB_x)\,\hat{k}$$

Resolve $M \longrightarrow \hat{i}, \hat{j}, \hat{k}$

$$\vec{M} = \vec{d} \times \vec{F}$$

$$\left.\begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \\ \sum M_{ox} = 0 \\ \sum M_{oy} = 0 \\ \sum M_{oz} = 0 \end{array}\right\} \begin{array}{c} \text{6 equations} \\ \downarrow \\ \text{6 unknowns} \end{array}$$



Ex: 3-D EQUILIBRIUM (FORCE AND MOMENT)

A UNIFORM BOOM (A-E) WEIGHS 0.5kN/m AND IS SUPPORTED BY CABLES AB and CD. VERTICAL LOAD (15kN ↓) CAN BE APPIED AT AN POSITION 'd' FROM E. E IS A BALL-SOCKET JOINT.

Find 'd' and support reactions at E.

GIVEN: A = (6,0,0) T_{CD} = 25kN

B = (0,-2,3)

C = (3,0,0)

D = (0,4,0)

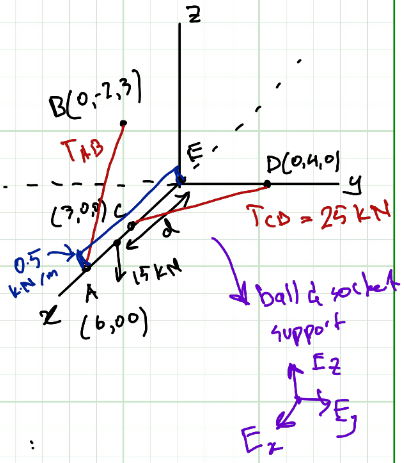
E = (0,0,0)

System:

Boom A-E Self Wgt (A-E) = 0.5kN/m

Cable A-B

Cable C-D = 25kN Load = 15kN



support rxn for ball & socket joint :

E_x , E_y , E_z

Unknown	Known
$d = ?$	$T_{CD} = 25$ kN
$E_x = ?$	UDL = ✓
$E_y = ?$	Point load
$E_z = ?$	
$T_{AB} = ?$	

Steps

1) Draw FBD
2) Resolve forces x, y, z
3) Calculate moments
4) apply eq. eq.

5 eq → 5 unknowns

$\vec{T}_{AB} = |T_{AB}|\,\vec{u}_{AB}$ A = (6,0,0) B = (0,-2,3)

$$\vec{u} = \frac{\vec{r}_{AB}}{|r_{AB}|} = \frac{(0-6)\hat{i} + (-2-0)\hat{j} + (3-0)\hat{k}}{\sqrt{(-6)^2 + (-2)^2 + (3^2)}}$$

$$\vec{T}_{AB} = -0.85714\,T_{AB}\,\hat{i} - 0.2857\,T_{AB}\,\hat{j} + 0.42857\,\hat{k}$$

$\vec{T}_{CD} = |T_{CD}|\,\vec{u}_{CD}$

$T_{CD} = 25$

C = (3,0;0)

D = (0,4,0)

$$\vec{u}_{CD} = \frac{\vec{r}_{CD}}{|r_{CD}|} = \frac{(0-3)\hat{i} + (4-0)\hat{j} + 0\hat{k}}{\sqrt{-3^2 + 4^2 + 0^2}}$$

$$\vec{T}_{CD} = 25\,\vec{u}_{CD} = -15\hat{i} + 20\hat{j} + 0\hat{k}$$

* UDL is 0.5 kN/m [↓] over 6 m of AE

$F_w = \left(0.5 \frac{kN}{\cancel{m}}\right)(6 \cancel{m}) = 3\ kN$ at 3 m

$|F_w| = 3\ N$ at $(3,0,0)$

$T_{AB} = ?$

$d = ?$

$\vec{F_w} = 0\hat{i} + 0\hat{j} - 3\hat{k}$ (acts at 3 m)

$\vec{F} = -15\hat{k}$ acts at $d = ?$

Take Moment at point E caz it gets rid of E_x, E_y, E_z

$+\circlearrowright \sum M_E = 0$

- $\sum M_E^x = 0$
- $\sum M_E^y = 0$
- $\sum M_E^z = 0$

* T_{AB} moment effect at E

$\vec{M}_E^{AB} = \vec{r}_{EA} \times \vec{T}_{AB}$

E→A

$\vec{r}_{::}$ = position vector from point E to anywhere along the line of action of force/cable T_{AB}

$$\vec{M}_E^{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (6-0) & (0-0) & (0-0) \\ -0.857T_{AB} & 0.2857T_{AB} & 0.42857T_{AB} \end{vmatrix}$$

$= 0\hat{i} - 2.5714T_{AB}\,\hat{j} - 1.7143T_{AB}\,\hat{k}$

- T_{CD} moment at E

$$\vec{M}_E^{CD} = \vec{r}_{EC} \times \vec{T}_{CD}$$

E → C

$$\vec{M}_E^{CD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ -15 & 20 & 0 \end{vmatrix} = $$

$$= 0\hat{i} - 0\hat{j} + 60\hat{k}$$

- W moment at E

$$\vec{M}_E^{FW} = \vec{r}_C \times \vec{F}_W$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 0 & 0 & -3 \end{vmatrix} = 0\hat{i} - (-9)\hat{j} + 0\hat{k}$$

- UDL moment at E

$$\vec{F}_E^{F=15} = r_{EF} \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d & 0 & 0 \\ 0 & 0 & -15 \end{vmatrix} = 0\hat{i} + 15\boxed{d}\hat{j} + 0\hat{k}$$

~~$\sum M_E^i = 0 =$~~ $\rightarrow$ not useful

solve for d

$\sum M_E^j = 0 = -2.5714 T_{AB} + 0 + 9 + 15d \rightarrow \boxed{d = 5.3999 \text{ m}}$ ✓

$\sum M_E^k = 0 \rightarrow \boxed{T_{AB} = 34.999 \text{ N}}$

$\sum F_x = 0 = 0.85714 T_{AB} - 15 + 0 + 0 + E_x \rightarrow E_x = 44.999 \text{ kN}$

$\sum F_y = 0 = -0.28571 T_{AB} + 20 + 0 + 0 + E_y \rightarrow E_y = -10 \text{ kN}$

$\sum F_z = 0 = -0.4285 T_{AB} + 0 - 3 - 15 - E_z \rightarrow E_z = 3.00 \text{ kN}$

Week 5: Lecture #1

- hinge rxn : table 5.2

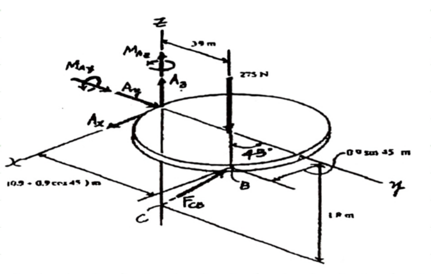
Eg 1: Circular Door

1.) Draw FBD : known forces
unknown forces
axes

2) Resolve F, M in x, y, z

3) Solve → $\underline{x}$ equations, $\underline{x}$ unknowns (max 6)

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0, \Sigma M_x = 0, \Sigma M_y = 0, \Sigma M_z = 0$$



$$\underbrace{\text{hinge has 5 unknowns : } A_x, A_y, A_z \; M_{ay}, M_{az}}_{\text{only allows rotation about } x\text{-axis}}$$

$\theta = 45°$

$F_g = 275$ N at G

$F_{CB} = ?$

$r = 0.9$m

$A = (0,0,0)$

$G = (0, 0.9, 0)$

$C = (0, 0, -1.8)$

B = x: $0.9 \sin 45°$
y: $0.9 + 0.9 \sin 45°$
z: 0

$B = (0.636, 1.536, 0)$

Resolving F_g

$\vec{F_g}: |F_g| \vec{u}_G = 275(-1\hat{k}) = -275\hat{k}$

$\vec{F_g} = -275\hat{k}$

Resolve strut $\vec{F}_{cb}$

$$\vec{F}_{CB} = |F_{CB}|\,\vec{u}_{CB}$$

$$\vec{u}_{CB} = \frac{r_B - r_C}{|r_{CB}|} = \frac{(0.636-0)\hat{i} + (1.536-0)\hat{j} + (0--1.8)\hat{k}}{\sqrt{(0.636)^2 + (1.536)^2 + (1.8)^2}}$$

$$= 0.2596\hat{i} + 0.627\hat{j} + 0.735\hat{k}$$

$$\therefore \vec{F}_{CB} = 0.2596|F_{CB}|\,\hat{i} + 0.627|F_{CB}|\,\hat{j} + 0.735|F_{CB}|\,\hat{k}$$

Find Moment about x, y, z by all forces

* taking moment about A takes out Ax, Ay, Az bc. no moment arm

F_g : $\vec{M}_A^{\vec{F}_g} = \vec{r}_{AG} \times \vec{F}_G$

$\vec{r}_{AG}$: from A to anywhere along line of action of F_g

$$= \vec{r}_G - \vec{r}_A$$

$$= (0\hat{i}, 0.9\hat{j}, 0\hat{k})$$

$$\vec{M}_A^{\vec{F}_g} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.9 & 0 \\ 0 & 0 & -275 \end{bmatrix}$$

$$= \left[0.9(-275) - 0(0)\right]\hat{i} - \left[0(-275) - 0(0)\right]\hat{j} + \Big[$$

$$\vec{M}_A^{\vec{F}_g} = -247.5\hat{i}$$

F_{CB} : $\vec{M}_A^{\vec{F}_{CB}} = \vec{r}_{AC} \times \vec{F}_{CB}$

$\vec{r}$ can be $\vec{r}_{AC}$ or $\vec{r}_{AB}$

$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A$

$$\vec{M}_A^{\vec{F}_{CB}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -1.8 \\ 0.2596|F_{CB}| & 0.627|F_{CB}| & 0.735|F_{CB}| \end{vmatrix} = 0\hat{i} + 0\hat{j} - 1.8\hat{k}$$

$$\vec{M}_A^{\vec{F}_{CB}} = 1.128|F_{CB}|\hat{i} - 0.467|F_{CB}|\hat{j} + 0\hat{k}$$

<u>Solve Eq. Equal.</u>

$\sum\vec{F} = 0$ and $\sum\vec{M} = 0$

$$\sum\vec{M}_A = \underbrace{M_A^{F_g} + M_A^{F_{CB}}}_{\text{applied forces}} + \overbrace{M_A^{y} + M_A^{z}}^{\text{hinge support}}$$

$$\sum\vec{M}_A^{\hat{i}} = 0 = \left(-243.5 + 1.1286|F_{CB}|\right)$$

$F_{CB} = 219.3\,N$

✓

$$\sum\vec{M}_A^{\hat{j}} = 0 = 0 - 0.467|F_{CB}| + M_{Ay} + 0$$

$M_{Ay} = 102.5\ N{\cdot}m$

$$\sum\vec{M}_A^{\hat{k}} = 0 = 0 + 0 + 0 + M_{Az}$$

$M_{Az} = 0$

$\Sigma F = 0$

$\Sigma F_x = 0 = 0.2596\,|F_{CB}| + A_x$

$A_x = -56.9\ N$

$\Sigma F_y = 0 = 0.627\,|F_{CB}| + A_y$

$A_y = -137.5\ N$

$\Sigma F_z = 0 = 0.735\,|F_{CB}| + A_z - 275$

$A_z = 113.8\ N$

Week 5: Lecture #3

Dot Product (3-D)

- cross product for 3-d moments

→ used to determine

(i) angle between vectors

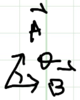
(ii) projection of a vector in a specified direction

Laws: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$$a(\vec{A}\cdot\vec{B}) = (a\vec{A})\cdot(\vec{B})$$

$$\vec{A}(\vec{B}+\vec{D}) = (\vec{A}\cdot\vec{B}) + (\vec{A}\cdot\vec{D})$$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$



$0° \leq \theta \leq 90°$

if $\theta = 0$, $\cos\theta = 1$

if $\theta = 90$, $\cos\theta = 0$

$$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x(\hat{i}\cdot\hat{i}) + A_xB_y(\hat{i}\cdot\hat{j}) + A_xB_z(\hat{i}\cdot\hat{k})$$

$$+ A_yB_x(\hat{j}+\hat{i}) + A_yB_y(\hat{j}\cdot\hat{j}) + A_yB_z(\hat{j}\cdot\hat{k})$$

$$+ A_zB_x(\hat{k}\cdot\hat{i}) + A_zB_y(\hat{k}\cdot\hat{j}) + A_zB_z(\hat{k}\cdot\hat{k})$$

$$\boxed{= A_xB_x + A_yB_y + A_zB_z} \leftarrow \text{scalar}$$

i·i = 1 i·j = 0 i·k = 0

j·i = 0 j·j = 1 j·k = 0

k·i = 0 k·j = 0 k·k = 1

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Exam-like 3-D Problem

Steps: 1. Draw FBD — known forces (applied and
└ unknown forces support rxn)

2. resolve F, M into x, y, z

3. apply equilibrium equ. to solve

Given:

$A = (0,0,0)$

$B = (2,0,0)$

$C = (2,-0.5,0)$

$D = (2,-0.5,2)$

$E = (2,-0.5,1.5)$

$F = (0,1,2.5)$

$G = (2,-0.5,0.5)$

Unknowns: A_x, A_y, A_z

D_x, D_y

T_{EF}

Approaches

- axis solution
  - ↳ moment about axis
- point solution
  - ↳ moment about point

→ **Resolving T_{EF}**

$$T_{EF} = |T_{EF}|\vec{u}_{EF} = T_{EF}(-0.743\hat{i} + 0.557\hat{j} + 0.371\hat{k})$$

take moment about AD axis

$$\rightarrow \quad \vec{u}_{AD} = \frac{(r_D - r_A)\hat{i} + (r_D - r_A)\hat{j} + (r_D - r_A)\hat{k}}{\sqrt{2^2 + 0.5^2 + 2^2}}$$

$$= 0.696\hat{i} - 0.174\hat{j} + 0.696\hat{k}$$

Take Moment about axis AD

↳ TEF will have moment about axis AD

↳ UDL will have moment about axis AD

$$\vec{M}_{AD} = \begin{array}{r} \\ \\ \text{point load} \rightarrow \end{array} \begin{vmatrix} 0.696 & -0.174 & 0.696 \\ 0 & 0 & -1.5 \\ 0 & -60 & 0 \end{vmatrix} \begin{array}{l} \leftarrow u_{AD} \\ \leftarrow \vec{r}_{DG} = -1.5\hat{k} \ \text{(distance)} \\ \leftarrow \text{force vector of UDL} \quad F_g = 60 \downarrow \end{array}$$

$$+ \begin{array}{r} \\ \\ \text{cable force} \rightarrow \end{array} \begin{vmatrix} 0.696 & -0.174 & 0.696 \\ 0 & 0 & -0.5 \\ -0.743T_{EF} & 0.557T_{EF} & 0.371T_{EF} \end{vmatrix}$$

$$= -0.696\left[(-60)(-1.5)\right] \leftarrow \text{point}$$

$$+T_{EF}\left[-0.696(0.557)(-0.5) + (0.174)(-0.5)(-0.743)\right] + 0$$

↖ cable

$$|T_{EF}| = 484.8 \text{ kN} \approx \boxed{485 \text{ kN}}$$

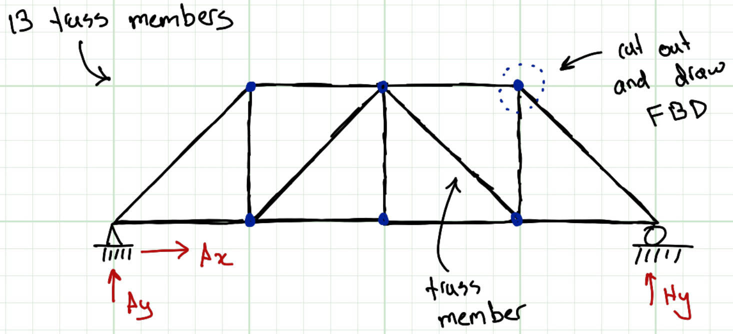
Point Approach Solution

$\Sigma M_A = 0$ M about A removes A_x A_y A_z

$$\vec{M}_A = \underbrace{\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -0.5 & 0.5 \\ 0 & -60 & 0 \end{bmatrix}}_{\text{point load}} + T_{EF}\underbrace{\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -0.5 & 1.5 \\ -0.743 & 0.557 & 0.371 \end{bmatrix}}_{\text{cable load}} + \underbrace{\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -0.5 & 2 \\ D_x & D_y & 0 \end{bmatrix}}_{\text{support}}$$

Week 6: Lecture #1 Trusses

- truss members can be in tension or compression
  - ↳ direction of force (T) - tension or (C) - compression

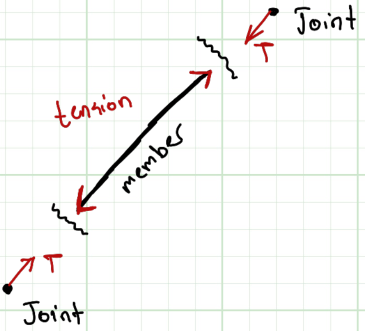


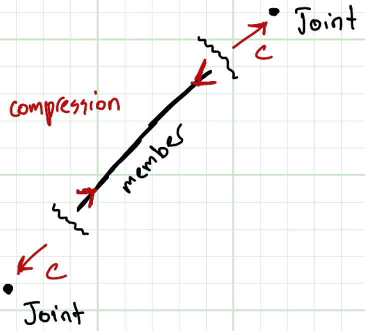
Trusses

- slender member joined at end points by pin-connections.
  - ↳ eg: bridges, roofs, stadium elements

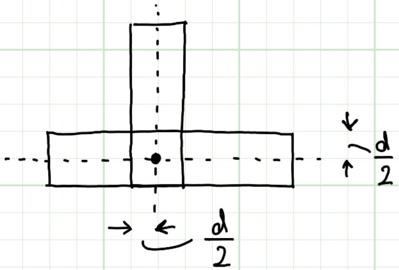
1. All loads are applied at joints

- self weight and thickness are typically minimal so are ignored
- pins = nodes (end of member) = joint
- member = link
- two force member (eighter compression or tension)





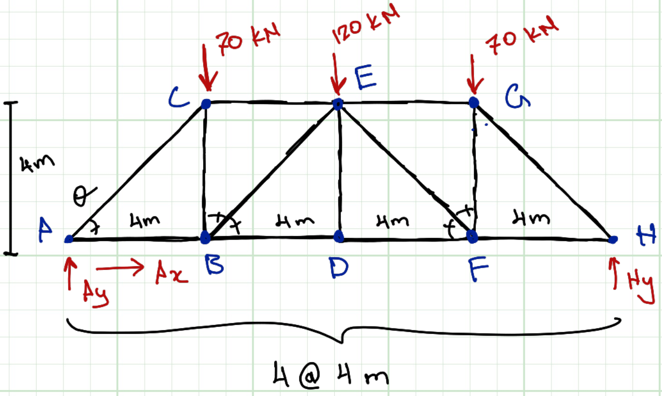
2. Joints/node/connection between truss members are smooth joints



2 methods

1. method of joints
2. method of sections

→ Eg: Method of Joints : truss equilibrium



Solve the forces in all the members

Steps

1. consider global/external equilibrium (Draw FBD)
2. solve rxn forces with eq. eq.
3. local equilibrium at the joints (2 unknowns)

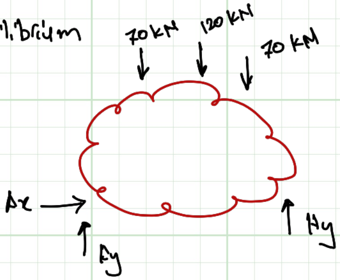
(3 unknowns)

repeat for all members

$\Sigma F_x = 0$

$\Sigma F_y = 0$

Global Equilibrium



→ Solve for Support Rxn Using Global Equilibrium

$\xrightarrow{+} \sum F_x = 0 = A_x$ $\boxed{A_x = 0}$

$+\circlearrowright \sum M_A = 0 = 70(4) + 120(8) + 70(12) - H_y(16)$

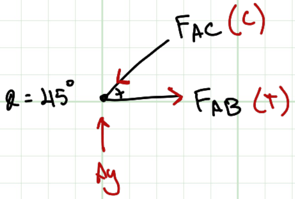
$\boxed{H_y = 130 \text{ kN}}$

$\uparrow^+ \sum F_y = 0 = A_y - 70 - 120 - 70 + 130$

$\boxed{A_y = 130}$

→ Local Equilibrium

Joint A



→ Resolve F_{AC} in x,y

$F_{AC_y} = F_{AC} \sin 45°$

$F_{AC_x} = F_{AC} \cos 45°$

$\uparrow \sum F_y = A_y - F_{AC} \sin 45 = 0$

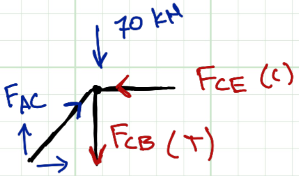
$130 - F_{AC} \sin 45 = 0$

$\boxed{F_{AC} = 183 \text{ kN (C)}}$

$\xrightarrow{+} \sum F_x = 0 = F_{AB} - F_{AC} \cos 45°$

$\boxed{F_{AB} = 130 \text{ kN (T)}}$

Joint C



$$+\uparrow \Sigma F_y = 0 = F_{AC}\cos 45^\circ - 70 - F_{CB}$$

$$183\cos 45^\circ - 70 - F_{CB} = 0$$

$$\boxed{F_{CB} = 60.0\text{ kN (T)}}$$

$$\xrightarrow{+} \Sigma F_x = 0 = F_{AC}\cos 40^\circ - F_{CE}$$

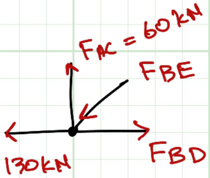
$$183\cos 40^\circ - F_{CE} = 0$$

$$\boxed{F_{CE} = 130\text{ kN (C)}}$$

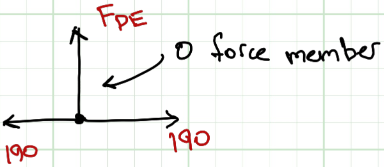
Joint B

$$\boxed{F_{BE} = 84.9\text{ kN (C)}}$$

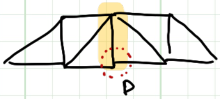
$$\boxed{F_{BD} = 190\text{ kN (T)}}$$

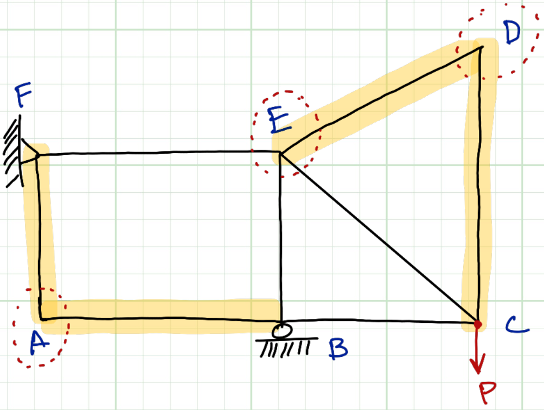


Joint D

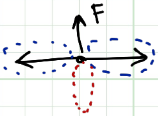


Zero Force Members





Joint D



$\Sigma F_x = 0$

$\Sigma F_y = \uparrow = 0$

$\therefore F = 0$

Joint D

$\Sigma F_x = 0 = -F_{DEx}$

$\Sigma F_y = 0 = -F_{DEy} - F_{CD}$

$\therefore F_{DE} = 0$

$F_{CD} = 0$

Joint A

$\Sigma F_x = 0 = F_{AB}$

$\Sigma F_y = 0 = F_{AF}$

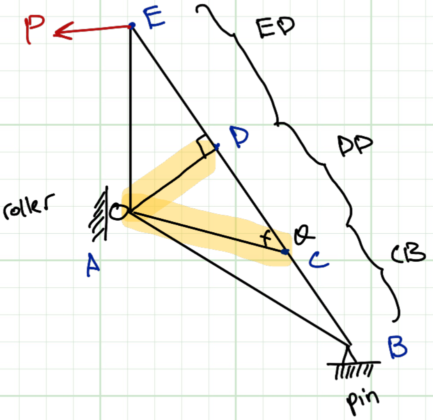
Rules of Thumb

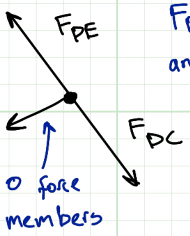
- if only 2 members that form truss joint, the 2 members can be zero force members

  If no support at joint and no applied forces at joint

- if 3 members forming truss joint And 2 of those members are colinear, the third member is a zero force member

  If no support at joint and no applied forces at joint

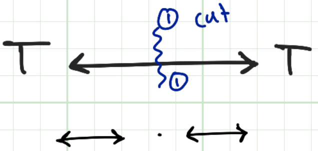


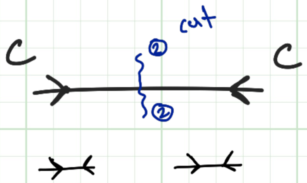


F_{DE} must equal and cancel out F_{DC}

Method of Sections

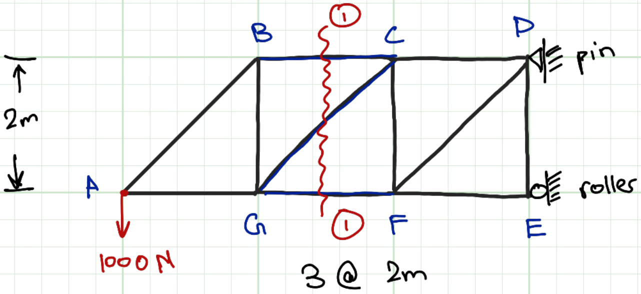
- basis is that the entire truss is in equilibrium, then any segment is also in equilibrium
- section a truss by taking imaginary cuts through it to expose the internal or truss members.

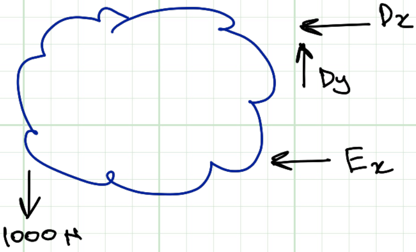




→ Eg: find F_{BC}, F_{GC}, F_{GF}

1. inspect for zero force mem.
2. global/global equilibrium





Solve Support Rxn

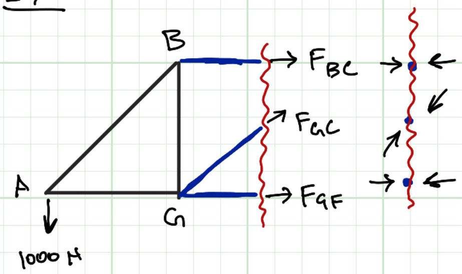
$+\circlearrowright \sum M_D = 0 = -(1000)(6) + E_x(2) \quad \cdots \quad E_x =$

$\xrightarrow{+} \sum F_x = 0 = -E_x - D_x \quad \cdots \quad D_x =$

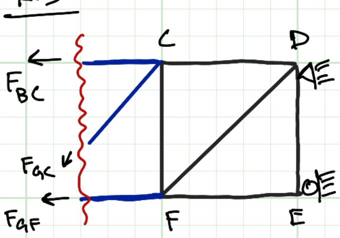
$+\uparrow \sum F_y = 0 = D_y - 1000 \quad \cdots \quad D_y =$

FBD at cut ① ①

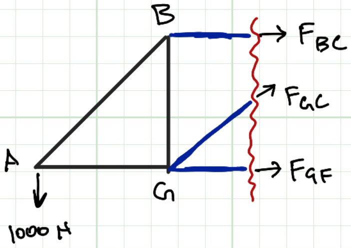
L.S.



R.S



* assuming they're in tension



Diagonal Members

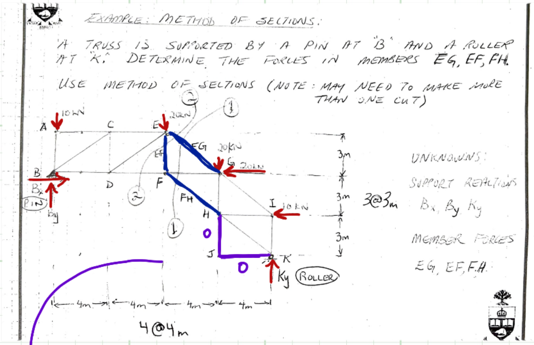
- resolve $\vec{F}_{GC}$ in x,y
- $\Sigma M_A = 0$

  $\Sigma F_x = 0$

  $\Sigma F_y = 0$

week 6: lecture #3

Truss Exam Problem



EXAMPLE: METHOD OF SECTIONS:

"A TRUSS IS SUPPORTED BY A PIN AT "B" AND A ROLLER AT "K." DETERMINE THE FORCES IN MEMBERS EG, EF, FH.

USE METHOD OF SECTIONS (NOTE: MAY NEED TO MAKE MORE THAN ONE CUT)

UNKNOWNS:

SUPPORT REACTIONS

Bx, By, Ky

MEMBER FORCES

EG, EF, F.H.

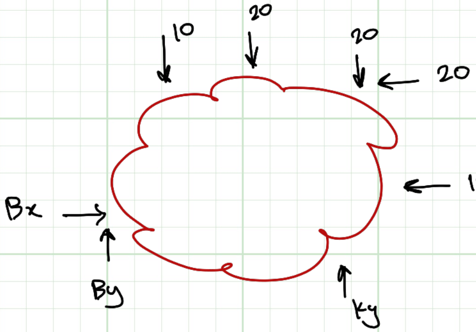
steps

1. inspect type sol
2. Draw FBD of global equilibrium (not show internal forces ... trusses)
3. solve unknowns (rxn forces)
4. look at internal equilibrium.

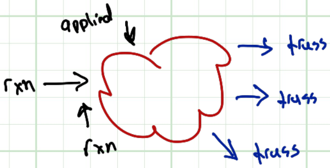
no rxn or applied forces at joint J, so 0-F member

cut section so that it goes through members of interest (Q is asking us)

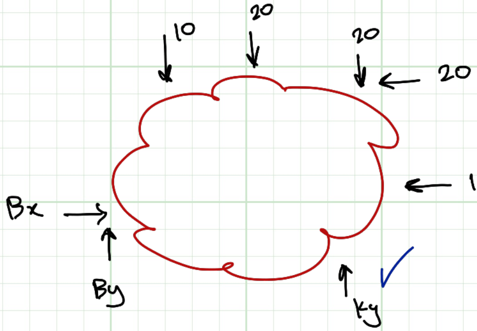
↳ L.S or R.S. draw FBD and expose forces (draw things going out)



3 unknowns ⇒ 3 eq.eq.



Step 1 **Support Rxn : global equ.**



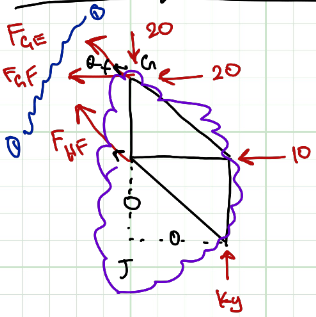
$$+\curvearrowright \sum M_B = 0 = 20(8) + 20(12) + 10(3) - k_y(16)$$

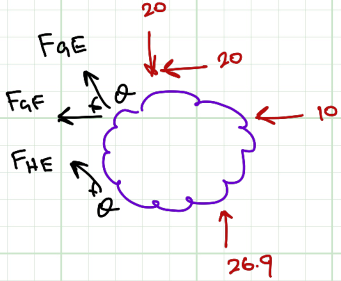
$$\boxed{\therefore k_y = 26.9\,kN \uparrow}$$

$$\xrightarrow{+} \sum F_x = 0 = B_x - 20 - 10 \qquad \boxed{\therefore B_x = 30\ kN \rightarrow}$$

$$\uparrow^{+} \sum F_y = 0 = B_y - 10 - 20 - 20 + k_y \checkmark \qquad \boxed{\therefore B_y = 23.1\ kN \uparrow}$$

Step 2. **Internal Equ: internal forces** R.H.S. cut 1.1





Resolve Forces at angles

$\boxed{HF}$ $\theta = 36.87°$

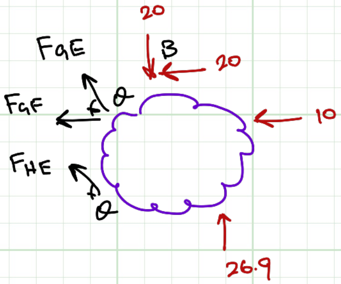
$F_{HF\,x} = \cos 36.87$

$F_{HF\,y} = \sin 36.87$

$\boxed{GE}$ $\theta = 36.87$

$F_{GE\,x} = \cos 36.87$

$F_{GE\,y} = \sin 36.87$



$$+\circlearrowleft \sum M_B = 0 = F_{HF}\cos 36.87(3) + 10(3) - 26.875(4)$$

$$\boxed{\therefore\ F_{HF} = 32.3\ \text{kN}\ (T)}$$

$$+\uparrow \sum F_x = 0 = F_{GE}\sin 36.87 + \underbrace{32.292\sin 36.87}_{\text{truss}} \underbrace{-20}_{\text{applied}} \underbrace{+26.875}_{\text{support}}$$

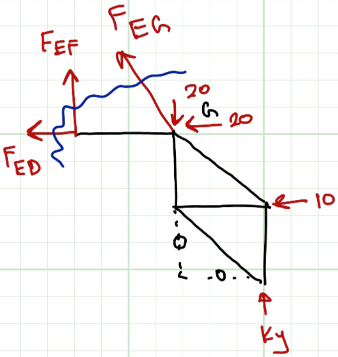
$$\boxed{\begin{aligned}\therefore\ F_{GE} &= -43.75\ (T)\\ &= 43.75\ (C)\end{aligned}}$$

$$\rightarrow^{+} \sum F_y = 0 = -F_{GF} - 20 - F_{GE}\cos 36.87^\circ - F_{HE}\cos 36.87^\circ - 10$$

$$\boxed{\begin{aligned}\therefore\ F_{GF} &= -20.8\ \text{kN}\ (T)\\ &= 20.8\ \text{kN}\ (C)\end{aligned}}$$

Cut 22

$+\circlearrowleft \sum M_G = 0 =$



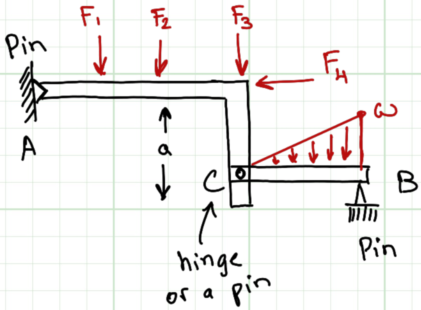
$$+\circlearrowleft \sum M_G = 0 = -26.9(4) + 10(3) + F_{EF}(4)$$

$$\boxed{\therefore F_{EF} = 19.40 \text{ kN (T)}}$$

Week 7: Lecture #1

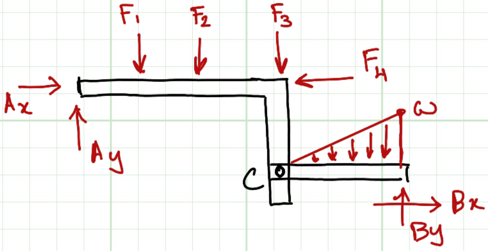
Frames + Machines (2-D)

- more complex than truss members
- forces are not carried axially in frames
- often comprised of pin-connected, multiforce members
- need to analyze pieces/sections if <u>unable to analyze the entire structure</u>
  - ↳ unable to solve with 3 eq. eq.
  - ↳ unknowns > 3 (2-d problem)



Frame → <u>there is a hinge (pin)</u> at point C

↳ internal pin or internal hinge



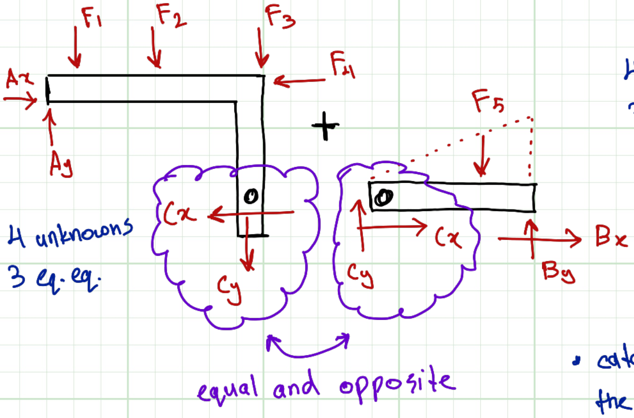
<u>4 unknowns</u>

Ax, Ay, Bx, By

- only express internal forces when you cut the system

Let's sever our system

↓



4 unknowns
3 eq. eq.

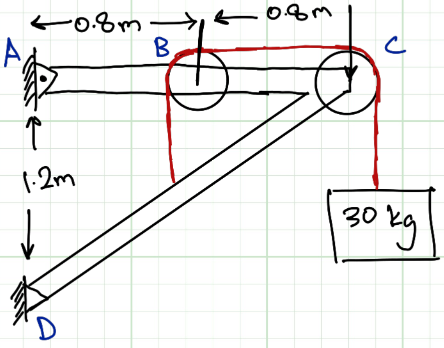
- catch is that 2 of the unknowns in each of the FBD's are the same

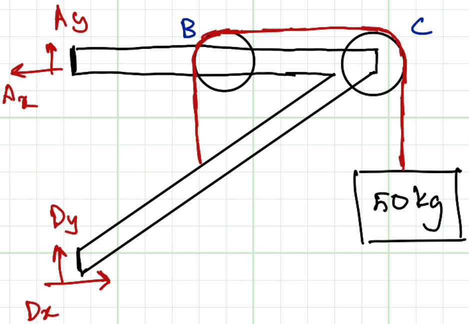
$$A_x, A_y, \boxed{C_x, C_y} \qquad \boxed{C_x, C_y}, B_x, B_y$$

$$\underbrace{A_x, A_y, C_x, C_y, B_x, B_y}_{}$$

in reality, we actually have 6 unknowns and 3+3 = 6 eq. eq.

→ **Eg:** Frame with Pulley : Determine horizontal and vertical components of the rxn at A and D. Pulley radius: 0.1 m



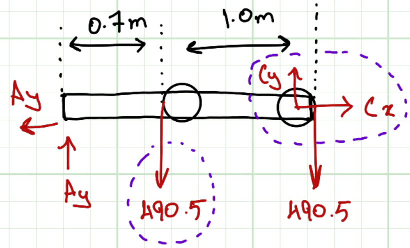


External FBD

assume directions of Ax, Ay, Dx, Dy

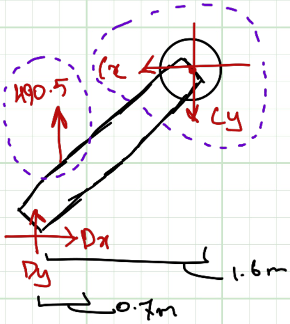
← $50 \times 9.81 = 490.5$ N

member ABC



equal and opposite

Member DC



Consider Equilib. of ABC

$$+\circlearrowleft \Sigma M_A = 0 = 490.5(0.7) + 490.5(1.7) - C_y(1.6)$$

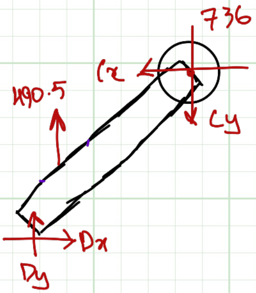
$$\boxed{C_y = 736\ \text{N}}$$

$$\xrightarrow{+} \Sigma F_x = 0 = -A_x + C_x \qquad \boxed{A_x = C_x}$$

$$\uparrow + \Sigma F_y = A_y - 490.5 - 490.5 + C_y$$ ✓

$$\boxed{A_y = 245\ \text{N}}$$

Consider Equilib. <u>DC</u>



$$+\circlearrowleft \sum M_D = 0 = -490.5(0.7) + \overset{736}{C_y}(1.6) - C_x(1.6)$$

$$\boxed{C_x = 695\ N}$$

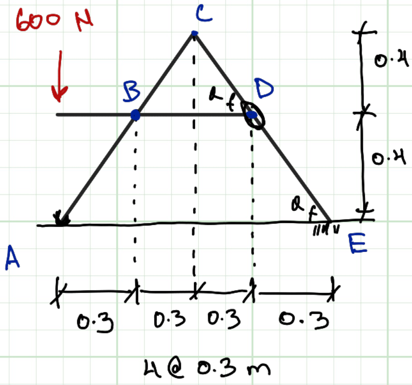
From **ABC** $\sum F_x = 0$, we got $A_x = C_x$

$$\boxed{\therefore A_x = 695\ N}$$

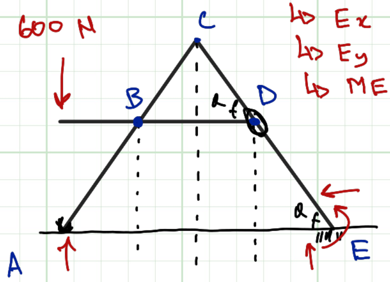
$$\xrightarrow{+} \sum F_x = D_x - C_x = 0 \qquad \boxed{D_x = 695\ N}$$

$$\uparrow^{+} \sum F_y = D_y + 490.5 - C_y = 0 \qquad \boxed{D_y = 245\ N}$$

→ Eg: Find reactions at fixed support E and smooth support A.
Pin attached B-D passes through smooth slot at D



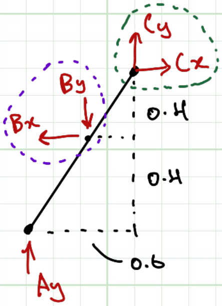
Global → 2D with 4 unknowns
↳ Ay
↳ Ex
↳ Ey
↳ ME



FBD A-C $\boxed{1}$

FBD BD $\boxed{2}$

FBD C-E $\boxed{3}$



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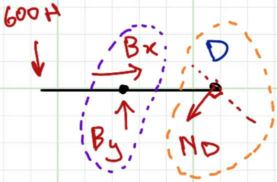

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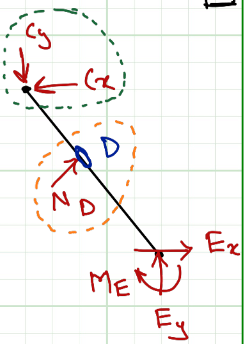
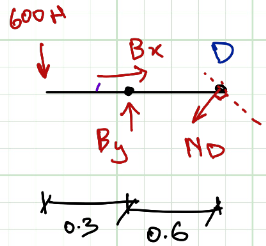


5 unknowns

3 unknowns

6 unknowns

FBD BD $\boxed{2}$



$$+\circlearrowright \Sigma M_B = 0 = -600(0.3) + N_D \cos 53.13(0.6)$$

$$N_D = 500\ N$$

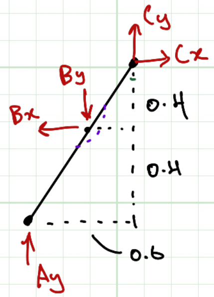
$$\xrightarrow{+} \Sigma F_x = B_x - N_D \sin 53.13 = 0$$

$$B_x = 400\ N$$

$$+\uparrow \Sigma F_y = -600 + B_y - N_D \cos 53.13$$

$$B_y = 900\ N$$

FBD $\boxed{1}$



$$+\circlearrowright \Sigma M_C = 0 = -\overset{900}{B_y}(0.3) + \overset{400}{B_x}(0.4) + A_y(0.6)$$

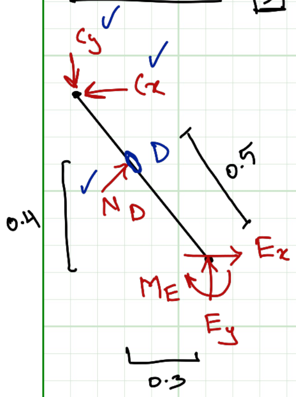
$$A_y = 183.3\ N$$

$$+\uparrow \Sigma F_y = 0 = A_y + C_y - B_y$$

$$C_y = 717\ N$$

$$\xrightarrow{+} \Sigma F_x = 0 = -B_x + C_x \qquad C_x = 400$$

FBD C-E $\boxed{3}$



$$+\circlearrowright \Sigma M_E = 0 = M_E - C_x(0.8) - C_y(0.6) + N_D(0.5)$$

$$M_E = 500\ N\cdot m$$

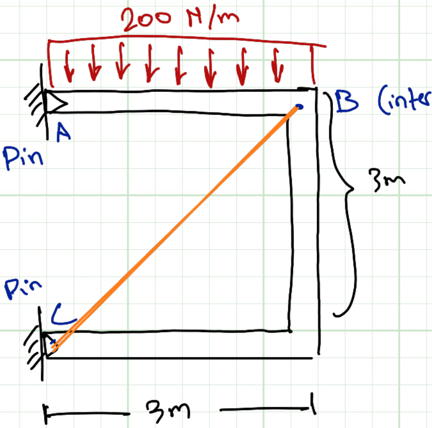
$$\xrightarrow{+} \Sigma F_x = E_x + N_D \sin 53.13 - C_x$$

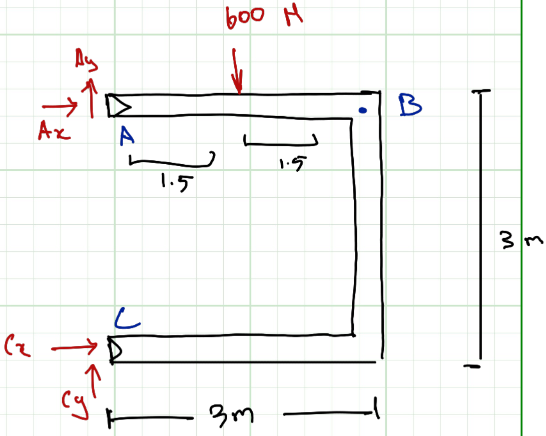
$$E_x = 0$$

$$+\uparrow \Sigma F_y = E_y + N_D \cos 53.13 - C_y$$

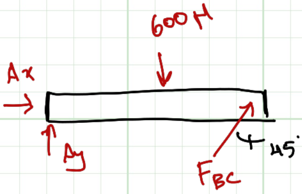
$$E_y = 417\ N$$

→ **Eg 2:** 2 force members ≡ force applied at only 2 points

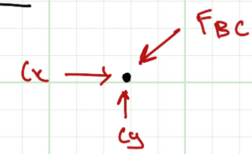




Member AB



Point C



AB

$+\circlearrowright \sum M_A = 600(1.5) - F_{BC}\cos 45(3) = 0$ $F_{BC} = 424$ N

$\xrightarrow{+} \sum F_x = A_x + F_{BC}\sin 45$ $A_x = -300$ N

$\uparrow^{+} \sum F_y = A_y + F_B \cos 45 - 600$ $A_y = 300$ N

$\xrightarrow{+} \sum F_x = C_x - (F_{BC})_x = 0$ $C_x = 300$ N

$\uparrow^{+} \sum F_y = C_y = 300$ N

Week 7: Lecture #3

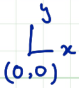
Chapter 9 : Center of Gravity and Centroids

→ **centroid:**

- location of a geometric center of a body (coincides with the centre of gravity I.A.O.I the element is homogenous (i.e. material)

- extra effort and consideration are required for non-homogenous or composite sections/materials

→ **Centroid of a line:** $(\bar{x}, \bar{y})$ ← coordinate (set an x,y-axes)



$$\boxed{\bar{x} = \frac{\Sigma \tilde{x}_i L_i}{L}} \qquad \boxed{\bar{y} = \frac{\Sigma \tilde{y}_i L_i}{L}}$$

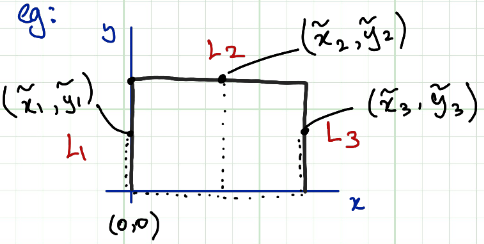
→ **centroid of an area:** $(\bar{x}, \bar{y})$ ← coordinate (set an x,y-axes)

$$\boxed{\bar{x} = \frac{\Sigma \tilde{x}_i A_i}{A}} \qquad \boxed{\bar{y} = \frac{\Sigma \tilde{y}_i A_i}{A}}$$

→ **centroid of a Volume** $= \frac{\text{mass}}{\text{density}}$ $(\bar{x}, \bar{y}, \bar{z})$ ← coordinate (set x,y,z axes)

$$\boxed{\bar{x} = \frac{\Sigma \tilde{x}_i V_i}{V}} \qquad \boxed{\bar{y} = \frac{\Sigma \tilde{y}_i V_i}{V}} \qquad \boxed{\bar{z} = \frac{\Sigma \tilde{z}_i V_i}{V}}$$

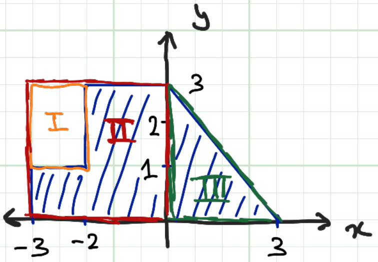
- Volum = $\frac{\text{mass}}{\text{density}}$ ← "ρ" ... some materials have different densities eg:



$$\bar{x} = \frac{\tilde{x}_1 L_1 + \tilde{x}_2 L_2 + \tilde{x}_3 L_3}{L}$$

$$\bar{y} = \frac{\tilde{y}_1 L_1 + \tilde{y}_2 L_2 + \tilde{y}_3 L_3}{L}$$

→ Eg: Composite body ... find centroid of plate



Steps

1. set x, y axes, portions (...i)
2. calculate subareas, sub-centroids $(A_i, \tilde{x}, \tilde{y})$, total area A.
3. calculate centroid of plate $(\bar{x}, \bar{y})$

(-) bc. it's a void whose are we will subtract

Portion	$A_i\ (m^2)$	$\tilde{x}_i$	$\tilde{y}_i$	$(\tilde{x}_i A_i)$	$(\tilde{y}, A_i)$
void I	$-[1 \times 2] = -2$	-2.5	2	$-2.5(-2) = 5$	$2(-2) = -4$
▨ II	$3 \times 3 = 9$	-1.5	1.5	$-1.5(9) = -13.5$	$1.5(9) = 13.5$
◺ III	$\frac{1}{2} \times 3 \times 3 = 4.5$	1	1	$1(4.5) = 4.5$	$1(4.5) = 4.5$

$\sum A_i = -2 + 9 + 4.5 = 11.5\ m^2$

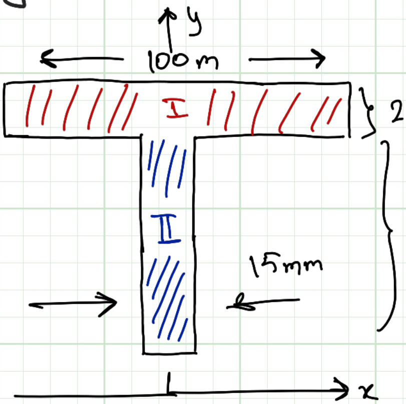
$\sum \tilde{x}_i A_i = -4.0\ m^3$ $\quad \sum \tilde{y}_i A_i = 14\ m^3$

$$\bar{x} = \frac{\sum \tilde{x}_i A_i}{A} = \frac{-4\ m^3}{11.5\ m^2} = 0.349\ m$$

$$\boxed{(\bar{x}, \bar{y}) = (0.349, 1.22)\ m}$$

$$\bar{y} = \frac{\sum \tilde{y}_i A_i}{A} = \frac{14\ m^3}{11.5} = 1.22\ m$$

Eg: Find Centroid T-section (x-section)



$\bar{x} = 0$

$$\bar{y} = \frac{\Sigma \tilde{y}_i A_i}{A} = \frac{\overset{\tilde{y}_1}{160}\,\overset{A_1}{(2000)} + \overset{\tilde{y}_2}{75}\,\overset{A_2}{(2250)}}{100 \times 20 + 15(150)} = 115 \text{ mm}$$

centroid T-section (x-section) = $(\bar{x}, \bar{y}) = (0, 115)$ mm

Week 8: Lec #1

Ch. 11: Moment of Inertia (I) $[mm^4]$

= cross sectional property (I) = second moment of area

→ relates normal stress (σ) acting on the transverse section of a beam to the bending moment (elastic)

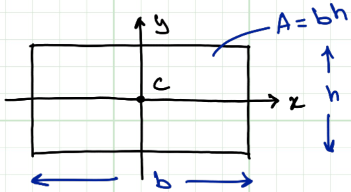
cross section

→ describes resistance to bending

$$\boxed{\sigma = \frac{My}{I}}$$

simple shapes: see back of Hibbeler + Panesar handout

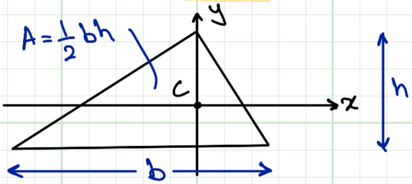
RECTANGULAR AREA



$$\boxed{I_x = \frac{1}{12} bh^3}$$

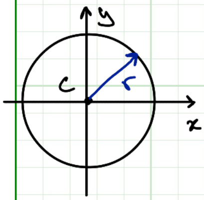
$$\boxed{I_y = \frac{1}{12} hb^3}$$

TRIANGULAR AREA



$$\boxed{I_x = \frac{1}{36} bh^3}$$

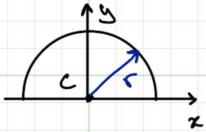
CIRCULAR AREA



$$\boxed{I_x = \frac{1}{4} \pi r^4}$$

$$\boxed{I_y = \frac{1}{4} \pi r^4}$$

SEMI-CIRCULAR AREA



$$\boxed{I_x = \frac{1}{8} \pi r^4}$$

$$\boxed{I_y = \frac{1}{8} \pi r^4}$$

composites ⇒ parallel axis theorem

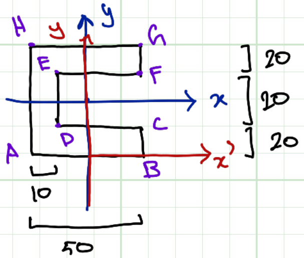
$$\boxed{\begin{aligned} I_x' &= I_x + Ad^2_y \\ I_y' &= I_y + Ad^2 x \end{aligned}}$$

d = distance from specified axis to centroid of sub-section

x' = composite

x = simple

→ Eg: calculate moment of inertia (I) about x-axis, x'-axis



1) identify portions

2) apply parallel axis theorem x-axis

x'-axis

x-axis

Portion	I_x	$d_y \sim$ "$\bar{y}$"	A	Ad_y^2	$=I_x+Ad_y^2$
ABGH	$\frac{1}{12}(50)(60)^3$	0	50 x60	0	90×10^4
CDEF (void)	$-\frac{1}{12}(40)(20)^3$	0	40 x 20	0	-2.66×10^4

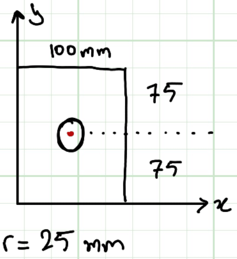
$$I_x = (90 \times 10^4) + (-2.66 \times 10^4) = 87.3 \times 10^4 \text{ mm}^4$$

x'-axis

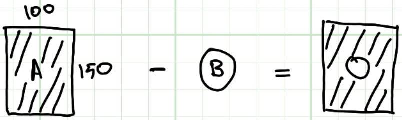
Portion	I_x	$d_y \sim$ "$\bar{y}$"	A	Ad_y^2	$=I_x+Ad_y^2$
ABGH	$\frac{1}{12}(50)(60)^3$	$60/2 = 30$	50 x60	270×10^4	$360(10)^4$
CDEF (void)	$-\frac{1}{12}(40)(20)^3$	$20 + \frac{20}{2} = 30$	40 x 20	$-72(10)^4$	$-74.7(10)^4$

$$I_x = (360 \times 10^4) + (-74.7 \times 10^4) = 285.3 \times 10^4 \text{ mm}^4$$

→ Eg 2: calculate I about x-axis



1) identify portions
2) apply parallel axis theorem



Portion A

$$I_{x'} = I_x + Ad_y^2$$

$$= \underset{\text{portion A}}{\left[I_x + Ad_y^{\,2}\right]} - \underset{\text{portion B}}{\left[I_x + Ad_y^2\right]}$$

$$= \left[\frac{1}{12}(100)(150)^3 + (100 \times 150)(75)^2\right]$$

$$- \left[\frac{1}{4}\pi(25)^4 + \pi(25)^2(75)^2\right]$$

composite shape $I_{x'} = I_A - I_B = 101.1 \times 10^6 \text{ mm}^4$

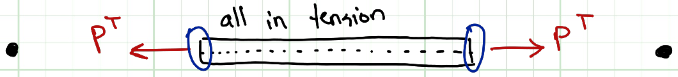
Week 8

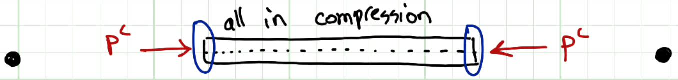
Complementary Notes

→ 1.1, 1.2
→ 2.1, 2.2, 2.3
→ 3.1, 3.2, 3.4

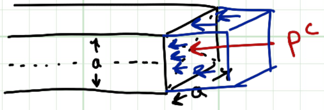
Chapter 1

axial σ, ε : Truss Member





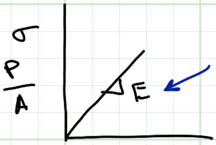
Cross Section



A : a×a
F: P^c

$$\sigma = \frac{F}{A} = \frac{P^c}{a \cdot a} = \frac{kN}{m^2} = MPa$$

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$



$$\varepsilon = \frac{\Delta L}{L}$$

$$E = \frac{PL}{A\,\Delta L}$$

$$\Delta L = \frac{PL}{EA}$$

load/force
initial lenght

$$\boxed{\Delta L = \frac{PL}{EA}}$$

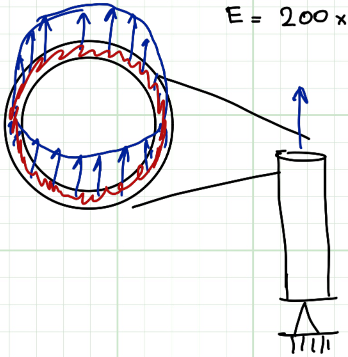
elastic modulus
cross sectional area

$$\boxed{\sigma = \frac{P}{A}}$$

$$\boxed{\varepsilon = \frac{\Delta L}{L}}$$

→ Eg: steel Pipe : axial tension
L of pipe = 4m
outer diameter = 101.6 mm
wall thickness = 5.76 mm
$E = 200 \times 10^3$ MPa

Determine axial stress that section can take and determine elongation



$$\sigma = \frac{F}{A} = \frac{150}{(\pi r_o^2) - (\pi r_i)^2}$$

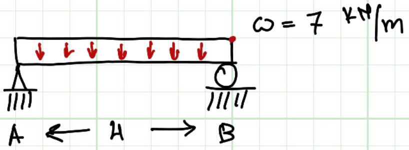
$$= \frac{150000}{\frac{\pi}{4}(10.16 - 90.12)^2}$$

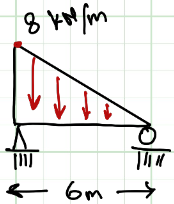
$$\boxed{\sigma = 86.7 \text{ MPa (tension)}}$$

$$\Delta L = \frac{PL}{EA}$$

$$= \frac{86.7\,(4000\text{mm})}{200 \times 10^3} = \boxed{\Delta L = 1.73 \text{ mm}}$$

Stress Blocks



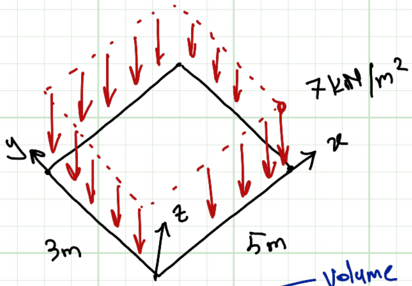


F = 7 × 4 = 28 kN

Acts where? 2m from A

$F = \frac{1}{2}(6)(8) = 24$ kN

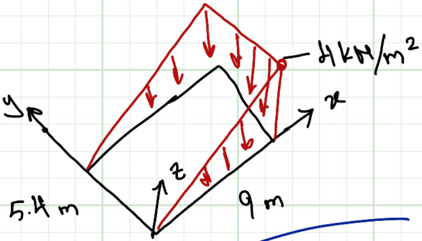
Acts: at 2m from A



$$\boxed{\frac{kN}{m^2} = \frac{\text{Force}}{\text{Area}} = \sigma = \text{Pressure or stress}}$$

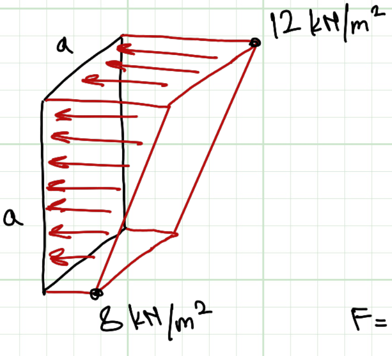
$$F = \overbrace{l \times w \times h}^{\text{volume stress block}} = 5m \times 3m \times 7kN/m^2 = [kN]$$

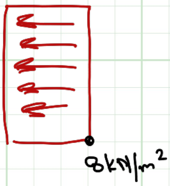
Acts = at centroid = $(2.5 \text{ m in } x, 1.5 \text{ m in } y)$



$$F: \overbrace{\frac{1}{2} l \times w \times h}^{\text{volume pressure block}} = \frac{1}{2}(9)(5.4)(4)$$

Acts: centroid $x = 6m$, $y = 2.7m$





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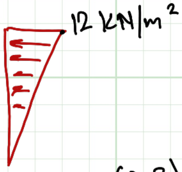


$$F = l \times w \times h$$
$$= a \times a \times 8$$

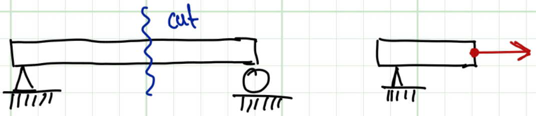
$$F = \frac{1}{2} l \times w \times \overset{(12-8)}{h}$$
$$= \frac{1}{2}(a \times a) \times 4$$

Week 9: Lecture # 1

Chapter 7: Internal Forces (7.1 - 7.3)

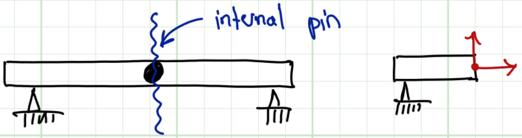
complementary (1.1, 1.2) ✓
Notes selected (2.1, 2.2, 2.3)
eventually (3.1, 3.2, 3.4)

TRUSS



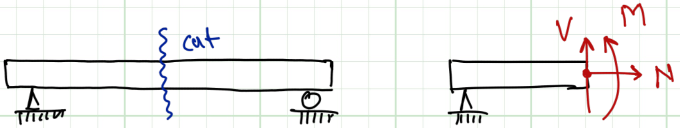
1 unknown

FRAME (PIN)



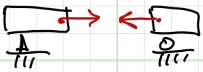
2 unknowns

BEAM



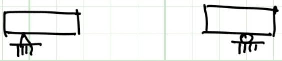
- the exposed (red) forces, or internal forces → assume direction
- For beams: (i) N: normal / axial force
  (ii) V: shear force
  (iii) M: bending moment force
- Forces in R.S. and L.S. Must Be Equal and Opposite

Truss

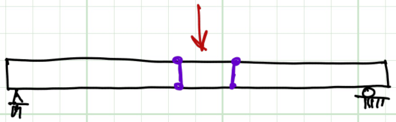


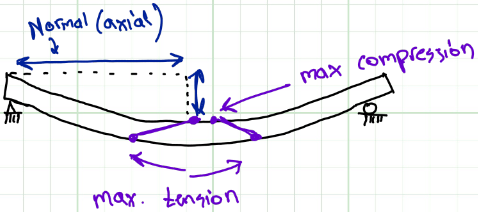
Frame (pin)

Beams

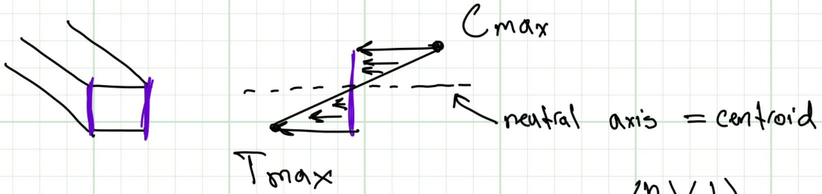


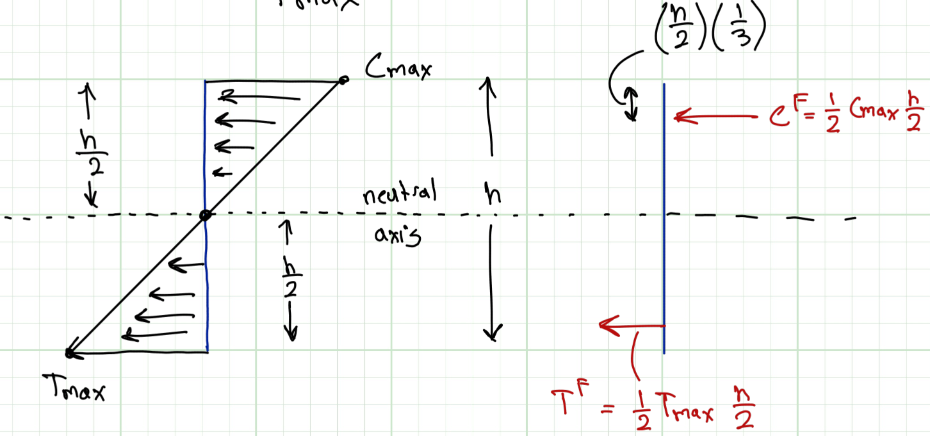
longitudinal (x-section) frame





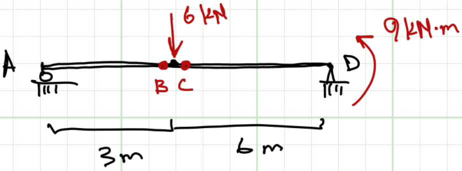
- stresses are neutral in the centroid of the cross section bc. it's the neutral axis

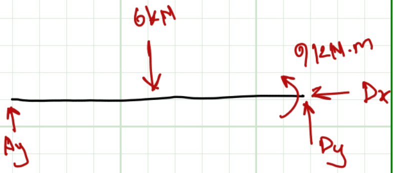




$$\boxed{M_o = C^F \times \frac{2}{3}\left(\frac{h}{2}\right) + T^F\left(\frac{2}{3}\right)\left(\frac{h}{2}\right)}$$

→ Eg: Determine the normal (axial) force (N), shear (V), momen (M) at B, C of point load





Step 1: Global Equilibrium to find support rxn (Ay, Dx, Dy)

Step 2: internal load / equilibrium

↳ cut member and expose (N, V, M)

Step #1

$\xrightarrow{+} \sum F_x = 0$ $\boxed{D_x = 0 \text{ kN}}$

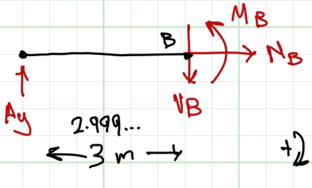
$+\uparrow \sum F_y = 0 = A_y - 6 + D_y$

$+\circlearrowleft \sum M_D = 0 = 9A_y - 6(6) - 9 = 0$

$\boxed{D_y = 1 \text{ kN}}$

$\boxed{A_y = 5 \text{kN}}$

Internal at B

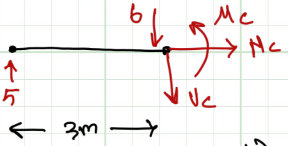


$\xrightarrow{+} \sum F_x = 0 = N_B$ $\boxed{N_B = 0}$

$+\uparrow \sum F_y = 0 = 5 - V_B$ $\boxed{V_B = 5 \text{ kN} \downarrow}$

$+\circlearrowleft \sum M_B = 0 = 5(3) - M_B$ $\boxed{M_B = 15 \text{ kN·m} \circlearrowleft}$

Internal at C



$\xrightarrow{+} \sum F_x = 0$ $\boxed{N_C = 0}$

$5 - (-1) = 6$
applied load

$+\uparrow \sum F_y = 0 = 5 - 6 - V_C$ $\boxed{V_C = -1 \text{ kN}}$

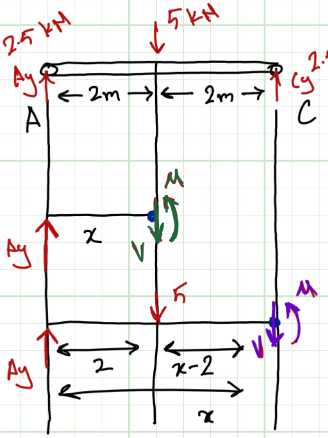
3.0001

$+\circlearrowleft \sum M_C = 5(3) - M_C = 0$ $M_C = 15 \text{ kN·m}$

* "6" has no moment at C

Week 9 : Lecture # 2

Draw Shear Force + Bending Moment Diagram of Beam



Cut locations

$0 < x < 2$

$2 < x < 4$

Steps

1. External equilibrium (support)
2. internal equilib. (internal forces V, M)

Step #1

$+\circlearrowright \sum M_A = 5(2) - 4C_y$ $\boxed{C_y = 2.5 \text{ kN}}$

$\uparrow^{+} \sum F_y = 0 = A_y - 5 + C_y$ $\boxed{A_y = 2.5 \text{ kN}}$

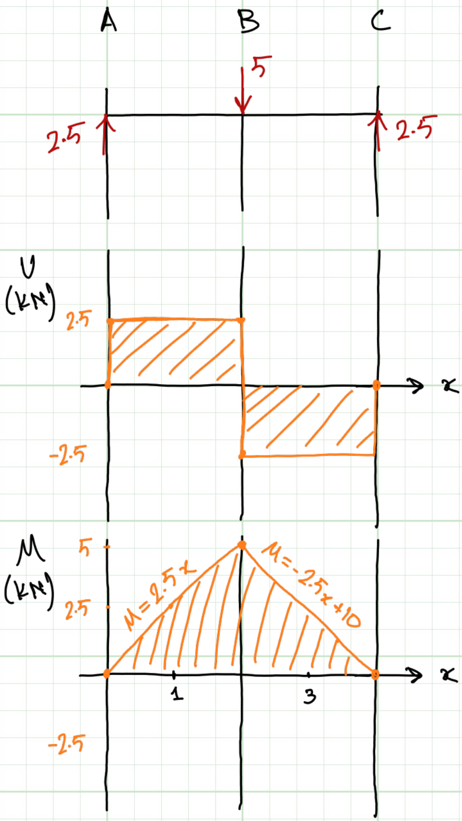
Step #2

$\boxed{0 < x < 2}$ $\uparrow^{+} \sum F_y = 0 = 2.5 - V$ $\boxed{V = 2.5 \text{ kN}}$

$+\circlearrowright \sum M_{\bullet} = A_y(x) - M = 0$ $\boxed{M = 2.5x}$

$\boxed{2 < x < 4}$ $\uparrow^{+} \sum F_y = 0 = 2.5 - 5 - V$ $\boxed{V = -2.5 \text{ kN}}$

$+\circlearrowright \sum M_{\bullet} = 0 = 2.5x - 5(x-2) - M$ $\boxed{M = -2.5x + 10}$



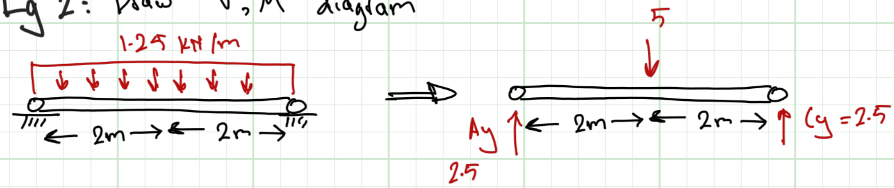
Relationships V, M

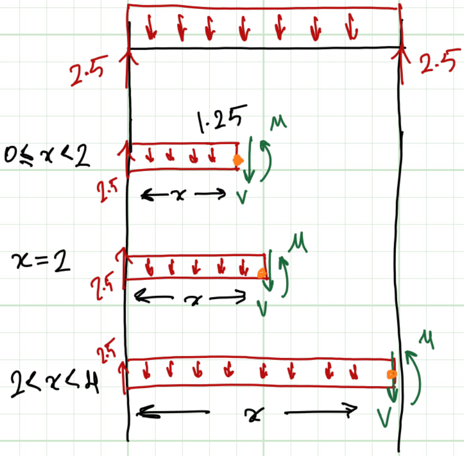
- Slope of moment $\boxed{\frac{dM}{dx} = V}$
- Change in Moment = Area under shear diagram

$$\boxed{\Delta M = \int V dx}$$

- Point of Zero shear = Minimum moment

→ Eg 2: Draw V, M diagram





Cut $0 \leq x < 2$

$\uparrow^+ \sum F_y = 0 = 2.5 - 1.25x - V = 0$

$\boxed{V = 2.5 - 1.25x}$

$+\circlearrowright \sum M = 0 = 2.5x - 1.25x\left(\frac{x}{2}\right) - M$

$\boxed{M = -0.625x^2 + 2.5x}$

Cut $x = 2$

$\uparrow^+ \sum F_y = 0 = 2.5 - 1.25(x) - V$

$\boxed{V = 1.25x - 2.5}$

Cut $2 < x < 4$

$\uparrow^+ \sum F_y = 0 = 2.5 \overbrace{-1.25(2) - 1.25(x-2)}^{UDL} - V$

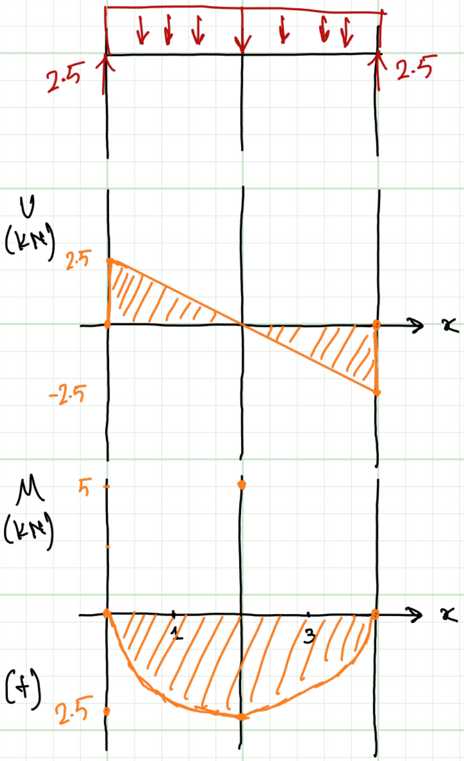
$\boxed{V = 2.5 - 1.25x}$

$+\circlearrowright \sum M = 2.5x - 1.25x\left(\frac{x}{2}\right) - M$

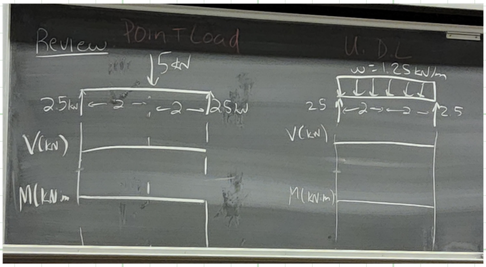
$\boxed{M = 2.5x - \frac{1.25}{2}x^2}$

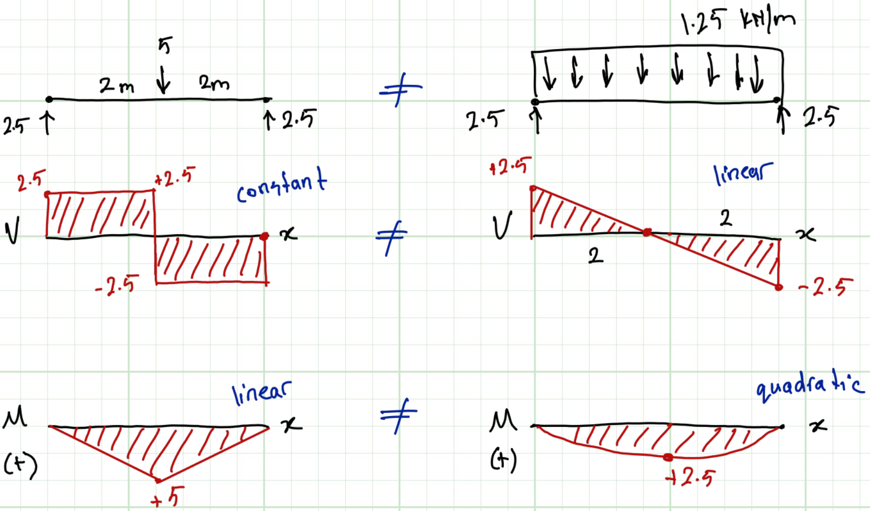
$+\circlearrowright \sum M = 2.5(x) - 1.25(x(x-2)) - 1.25(x-2)\left(\frac{x-2}{2}\right) - M$

$\boxed{M = 2.5x - 0.625x^2}$


2.5
2.5
V
(kN)
2.5
-2.5
x
M
(kN)
5
1
3
x
(+)
2.5

Week 9: Lecture #2



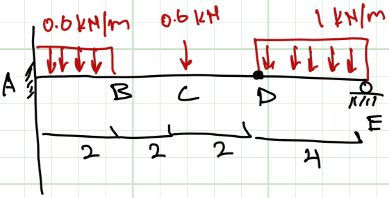


Properties

- Δ shear = area under loading
- slope of moment = shear $\frac{dM}{dx} = V$
- Δmoment = area under shear diagram

$$\Delta \text{moment} = \int V\, dx$$

Draw V, M Diagram (A = fixed roller, E = roller, D = internal pin)



1) Global Equilibrium

2) Internal equilibrium (cuts)

Left Side

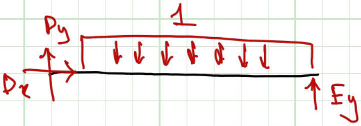
$+\circlearrowleft \sum M_A = 0$

$$\boxed{\begin{aligned} M_A &= -15.6 \text{ kN·m} \circlearrowleft \\ &= 15.6 \text{ kN·m} \circlearrowright \end{aligned}}$$

$\uparrow^{+} \sum F_y = 0$ $\boxed{A_y = 3.8 \text{ kN}}$

$\rightarrow^{+} \sum F_x = 0$ $\boxed{A_x = 0}$

Right Side

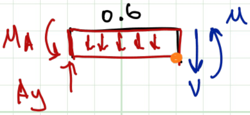


$+\circlearrowleft \sum M_D = 0$ $\boxed{E_y = 2 \text{ kN}}$

$\uparrow^{+} \sum F_y = 0$ $\boxed{D_y = 2 \text{ kN}}$

$\rightarrow^{+} \sum F_x = 0$ $\boxed{D_x = 0}$

$\boxed{0 \leq x < 2}$



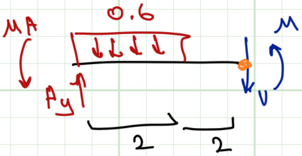
$\uparrow^{+} \sum F_y = 0 = 3.8 - 0.6x - V$

$$\boxed{V = -0.6x + 38}$$

$+\circlearrowleft \sum M = 0 = -15.6 + 3.8x - 0.6x\left(\frac{x}{2}\right) - M$

$$\boxed{M = -0.3x^2 + 3.8x - 15.6}$$

$\boxed{2 < x < 4}$

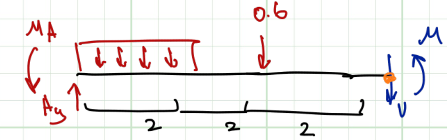


$\uparrow^{+} \sum F_y = 0$ $\boxed{V = 2.6 \text{ kN}}$

$+\circlearrowleft \sum M = 0$ $\boxed{M = 2.6x - 14.4}$

$$-15.6 + 3.8x - 0.6(2)\left(\frac{x}{2}\right)\left(x - \frac{2}{2}\right) - M = 0$$

$\boxed{4 < x < 6}$



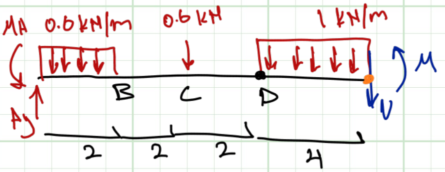
$$\uparrow^{+} \sum F_y = 0 \qquad 3.8 - 0.6(2) - 0.6 - V = 0$$

$$\boxed{V = 2.0 \text{ kN}}$$

$$+\circlearrowright \sum M_o = 0 = -15.6 + 3.8x - 0.6(2)\left(x - \frac{2}{2}\right) - 0.6(x-4) - M$$

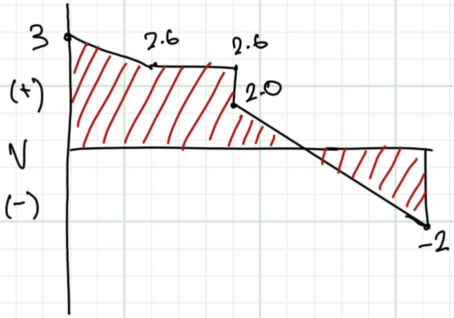
$$\boxed{M = 2.0x - 12}$$

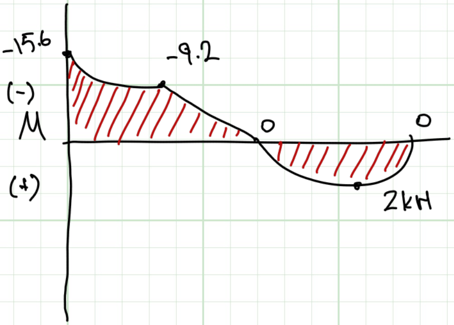
$\boxed{6 < x < 10}$



$$\uparrow^{+} \sum F_y = 0 \qquad \boxed{V = -x + 8}$$

$$\boxed{M = -\frac{x^2}{2} + 8x - 30}$$





Week 10, lec #1

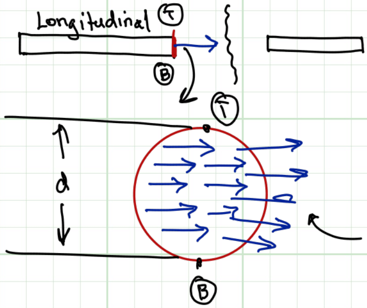
Load Pressure + Stress Blocks

complementary notes
2.1, 2.2, 2.3 and
3.1, 3.2, 3.4

Assumptions

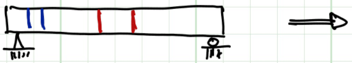
1. linear elastic
2. plane sections remain plane in bending
3. bending is independent of stresses caused by shear, normal forces

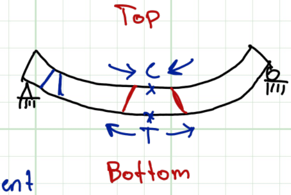
Truss

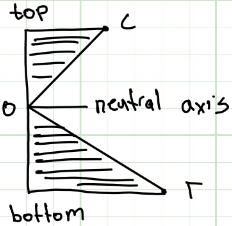


$$\sigma^F = \frac{F}{A}$$

Beams







max. moment

$$\sigma = \frac{My}{I}$$

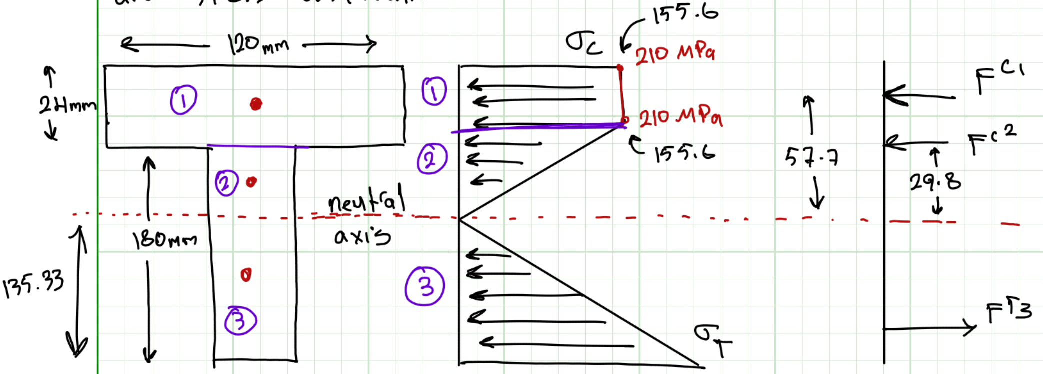
← geometry

← moment of inertia (resistance to bending)

$$S = \frac{y}{I}$$

← cross-section modulus

→ Eg 1 : Section has max C stress of 210 MPa, find max positive moment if factor of safety = 1.35. Given x-section and stress distribution



Centroid = neutral axis = $(\sigma = 0)$

$\sigma_c^{FS} = 210 \times 1.35$

$= 155.55$ MPa

$$\bar{y} = \frac{A_1\tilde{y}_1 + A_2\tilde{y}_2}{A_T}$$

$$= \frac{((20 \times 80)\cdot 90) + (120 \times 24)(180 + 12)}{(120 \times 180) + (120 \times 24)}$$

$\bar{y} = 135.33$mm

Region 1 F^{C1} = volume of stress-block

$= L \times w \times h$

$= 120 \times 24 \times 155.55$

$= 447981$ N $= \boxed{447.9 \text{ kN} = F^{①}}$

Region 2 F^{C2} = volume of stress-block

$= \frac{1}{2} \times L \times w \times h$

$= \frac{1}{2}(180 - 135.33)(155.55)(20)$

$\boxed{F^{②} = 69.48 \text{ kN}}$

Region 3 F^{T3} = volume of stress block

$$= \frac{1}{2} \times L \times w \times h$$

$$= \frac{1}{2} \times 20 \times 153.33 \times \sigma_T$$

$$= \frac{1}{2}(153.33)(20)\,\underline{\sigma_T}$$

$\xrightarrow{+} \sum F_x = 0$

$F_{C1} + F_{C2} = F_{T3}$

$F_{T3} = 517.46$

Find Moments

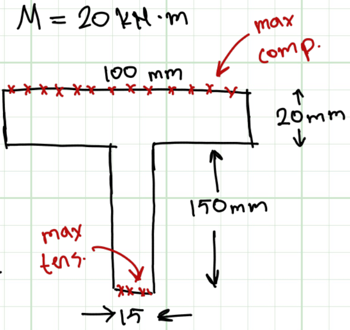
$$+\circlearrowleft \sum M_{\substack{\text{netral}\\ \text{axis}}} = F_{C1}y_{C1} + F_{C2}y_{C2} + F_{T3}y_{T3}$$

$$M_{max} = 74.6 \text{ kNm}$$

Week 10: Lecture #2

Scenario 1 (3.1)

Given $M = 20\ \text{kN}\cdot\text{m}$



Find:

$\sigma_{max\,C}$

$\sigma_{max\,T}$

Steps

① centroid

② find I

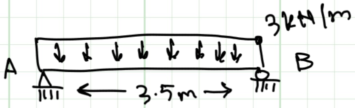
③ $\sigma = \frac{My}{I}$

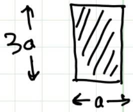
Scenario 2 (3.2)

Given: Load Factor = 2.0

$\sigma_{Tension} = 12\ \text{MPa} < \sigma^{comp}$

Find "a" or x-section geometry





Steps

① support

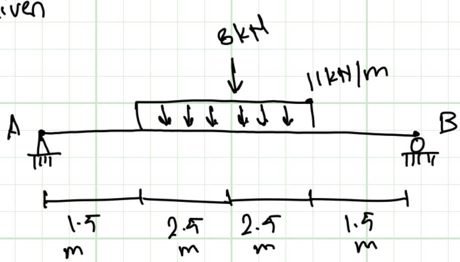
② max moment +(Bend. M. Diag)

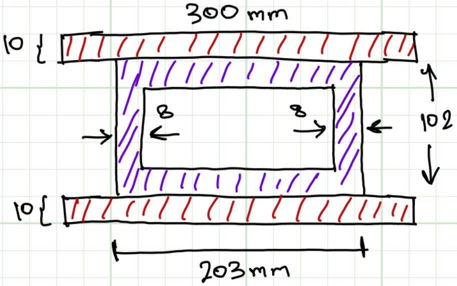
③ centroid, I, y

④ $\sigma = \frac{My}{I}$

Scenarion 3

Given





Find:

σ_c + location on beam

Steps

① support

② Mmax + location

③ I built-up section

④ $\sigma = \frac{My}{I}$

Beams

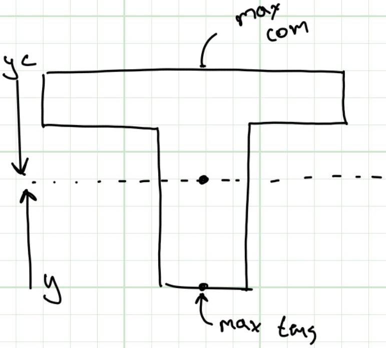
- bending → Moment + (internal)
  - ↳ centroids
  - ↳ I (internal)
  - ↳ $\boxed{\sigma = \frac{M\,y}{I}}$

internal F, M

MPa

x-section

$\sigma = \frac{F}{A}$ (MPa)



Senario # 1 (3.1)

$\bar{y}$ = 115 mm from bottom of web

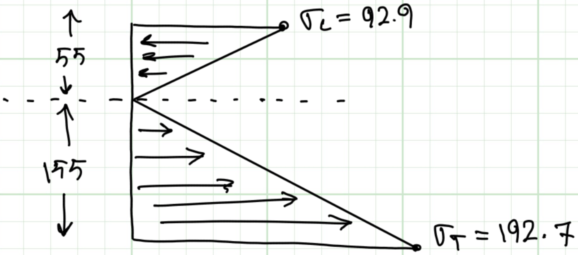
$$I = I_{flange} + I_{web}$$

$$= \left[\frac{1}{12}bh^3 + Ad^2\right] + \left[\frac{1}{12}bh^3 + Ad^2\right]$$

$$I = 11.94 \times 10^6 \text{ mm}^4$$

$$\sigma^c_{max} = \frac{My}{I} = \frac{20\times10^6 \times 55}{11.94\times10^6} = \boxed{92.9 \text{ MPa}}$$

$$\sigma^T_{max} = \frac{My}{I} = \frac{20\times10^6 \times 115}{11.94\times10^6} = \boxed{192.7 \text{ MPa}}$$

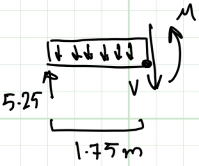


Scenario #2 (3.2)

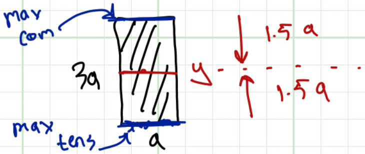
Support Rxns : $+\circlearrowleft \sum M_A = 0$ $B_y = 5.25$ kN ↑

$+\uparrow \sum F_y = 0$ $A_y = 5.25$ kN ↑

max moment of beam



$\boxed{M_o = 4.59 \text{ kN.m}}$ @ center of beam



$$\overset{\checkmark}{\sigma} = \frac{\overset{\checkmark}{M}\,\overset{\checkmark}{y}}{I}$$

Calculate I

$I_{rectangle} = \frac{1}{12} b h^3$ since $b = a$, $h = 3a$

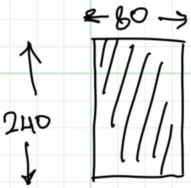
$= \frac{1}{13} a (3a)^3$

$I = 2.25 a^4$

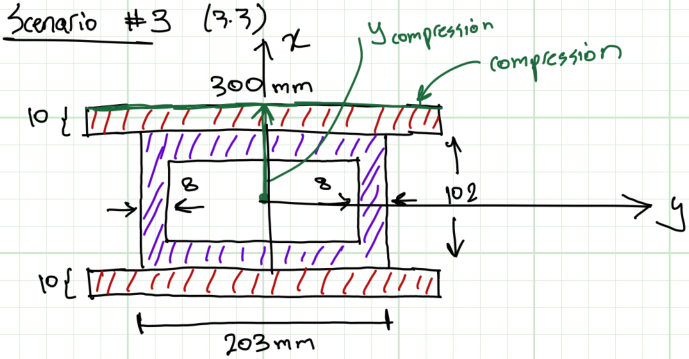
$$\overset{12}{\sigma} = \frac{\left(\overset{2}{\text{Load Factor}}\right) \times \overset{\text{found}}{M} \Big) y}{I}$$

$$12 = \frac{(2 \times 4.59)\, 1.5a}{2.25 a^4} \Longrightarrow a = 79.917 \text{ mm}$$

≈ 80 mm

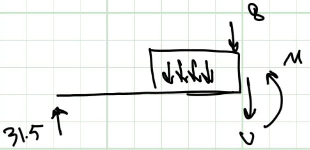


Scenario #3 (3.3)



1) $B_x = 31.5$ kN ↑

$A_x = 31.5$ kN ↑



$M_{max} = 91.6 \times 10^6$ N·mm

$$I = (I_y + Ad^2)^{Top} + (I_y + Ad^2)^{bottom} + I^{\square}$$

$$I = 26.406 \times 10^6 \text{ mm}^4$$

$$X = 51 + 10 \text{ mm} = 61 \text{ mm}$$

$$\sigma^c = \frac{M_{max}\, x_{max}}{I}$$

$$= 211.7 \text{ kN/mm}^2 = \boxed{212 \text{ MPa}} = \sigma_c^{max}$$

Week 10: Lec #3

Hydrostatics

$$\boxed{P = \rho g h = \gamma h}$$

$$\left(\frac{N}{m}\right)^2 = \left(\frac{kg}{m^3}\right)\left(\frac{m}{s^2}\right) m$$

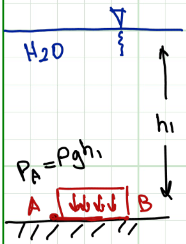
ρ = density

$g = 9.81$

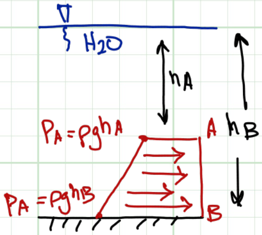
γ = self weight

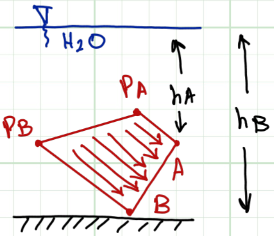
$\gamma = 1000$ kg/m^3 (H_2O)

h = depth from surface



$P_B = P_A$





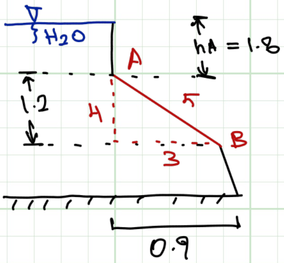
* 1) $P = \rho g h = \gamma h$

2) $P \propto h \rightarrow$ increases linearly

3) Pressure acts $\perp$ to submerged surface

Eg: What is the hydrostatic force on the gate + where it acts

$\gamma = \rho g = 10$ kN/m^3 = 10000 N/m^3, $w = 0.6$ m in/out page



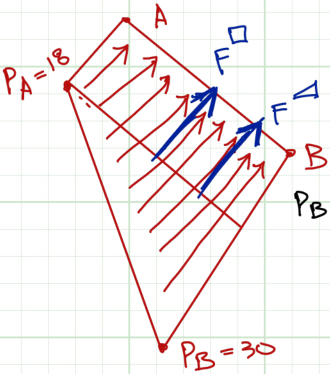
1) Draw hydrostatic pressure block

2) Calc P at A, B

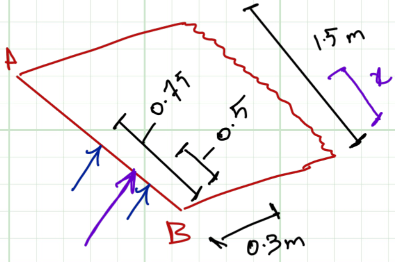
3) Calc force (from pressure $\Rightarrow$ volume of pressure block)

4) where does (F) act at?

$P_A = \rho g h = \rho g 1.8 = 18$



$P_B = 3\rho g$
$= 30$



$F_T = F^{\square} + F^{\triangleleft} = \boxed{21.6 \text{ kN}}$

$\Sigma M_B = 0 = 21.6(x) = F^{\square}(0.75) + F^{\Delta}(0.5)$

$\boxed{x = 0.688 \text{ m from B}}$

$F^{\square} = l \times w \times h$

$= 18 \times 0.6 \times 1.5$

$= \underline{16.2 \text{ kN}}$

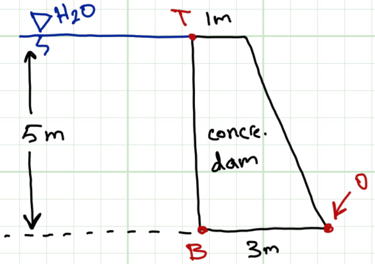
$F^{\Delta} = \frac{1}{2} \times l \times w \times h$

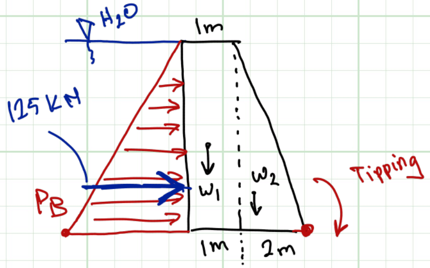
$= \frac{1}{2}(30 - 18)(1.5)(0.6)$

$= \underline{5.4 \text{ kN}}$

Week 11 : Lecture 1

Hydrostatics Eg: factor of safety (FS) against tipping of a tipping of a dam is $\left| \text{moment stabilizing} / \text{moment overturning} \right|$ about O. Determine FS if $\gamma_{concrete} = \rho g = 24 \text{ kN/m}^3$, $\gamma_{water} = \rho g = 10 \text{ kN/m}^3$





Steps

1) FBD (hydrostatic force), (stabilizing force) (concrete weight) or support
2) determing all the forces (taking pressure → forces)
3) Equilibrium : 3 Eq. Eq.

Water Pressure / Hydrostatic Pressure

- Pascal's Law : $P = \rho g h$ (pressure = rho · g · h)
- pressure is directly proport. to depth : $P \propto h$
- Pressure is ⊥ to submerged surface

- $P_T = \gamma h_1 = 0$
- $P_B = \gamma h_2 = 10 \times 5 = 50 \text{ kN/m}^2$

F = Volume under hydro-pressure block

* assume dam/water is 1 m in/out of page ... width = 1

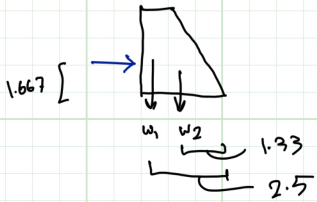
- $F = \frac{1}{2} \times b \times h \times \ell$

$= \frac{1}{2}(50)(5)\underline{(1)} = 125 \text{ kN}$

$$W_1 = \gamma_{concrete}\,(\text{Volume concrete})$$
$$= 24(5 \times 1m \times 1m) = 120 \text{ kN}$$

$$\boxed{\begin{array}{l} F = 125 \text{ kN} \\ W_1 = 120 \text{ kN} \\ W_2 = 120 \text{ kN} \end{array}}$$

$$W_2 = \gamma_{concrete}\,(\text{Volume } \Delta)$$
$$= 24\left(\frac{1}{2} \times 5 \times 2 \times 1\right) = 120 \text{ kN}$$



Factor of Safety

$$\boxed{FS = \left|\frac{M_{stabilizing}}{M_{overturning}}\right|}$$

$$\frac{(\text{contribution of concrete weight})}{(\text{contribution of water pressure})}$$

Moment Arms for $M_\bullet$

$$y_F = \frac{1}{3} \times 5 = 1.667 \text{ m (from } \bullet)$$

$$X_{w_1} = 2 + \frac{1}{2} = 2.5 \text{ m (from } \bullet)$$

$$x_{w_2} = \frac{2}{3} \times 2 = 1.33 \text{ m (from } \bullet)$$

Stabilizing M

$$+\circlearrowleft M_s = -W_1 x_{w_1} - W_2 x_{w_2}$$
$$= \underline{\underline{-459.996 \text{ kN}\cdot\text{m}}}$$

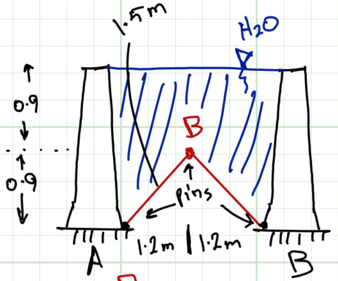
Overturning M

$$+\circlearrowleft M_o = F y_F$$
$$= 125(1.667)$$
$$= \underline{\underline{208.33 \text{ kN}}}$$

$$FS = \left|\frac{459.996}{208.33}\right| = \boxed{2.21 = FS \text{ against overturing}}$$

→ Eg 2: 2 Plates horizontally hinged at B. Find horizontal force per m of the channel length on each plate (A-B, B-C)

Plate = 250 kg



Steps

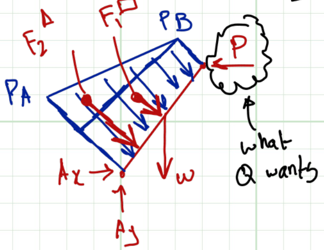
support

1) FBD ← weight, water

2) $P=\rho gh$, $P \propto h$, $P \perp$ surface

3) Equilibrium

since symmetric, only do 1 plate



$P_A = \rho g(1.8) = 1000(9.81)(1.8) = 17.66$ kN/m³

$P_B = \rho g(0.9) = 1000(9.81)(0.9) = 8.83$ kN/m³

Now convert to Forces

assume 1 m in/out page

* $F_1^{\square} = l \times w \times h = 8.829(1.5)\underline{(1.0)} = 13.24$ kN (per m)

* $F_2^{\Delta} = \frac{1}{2}(P_A - P_B)(1.5)(1.0) = 6.62$ kN (per m)

* $W = 250 \times 9.81 = 2.453$ kN

Solve for P (acting at B)

$$+\circlearrowleft\sum M_A = 0 = F^{\square}(0.75) + F^{\Delta}(0.5) + w(0.6) + P(0.9)$$

$P = 16.35$ kN/m

Week 11: Lecture #2

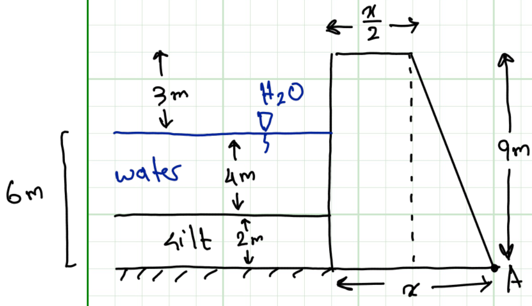
→ Eg: 9m tall gravity dam has unknown dimensions x. Draw FBD and its forces, then find x for a Factor of Safety (FS) of 1.75 against overturning about A. Given: $\rho_w = 1000$ kg/m³

$\rho_{silt} = 1760$ kg/m³

$\rho_{con} = 2400$ kg/m³

assume

$w = 1$



Steps

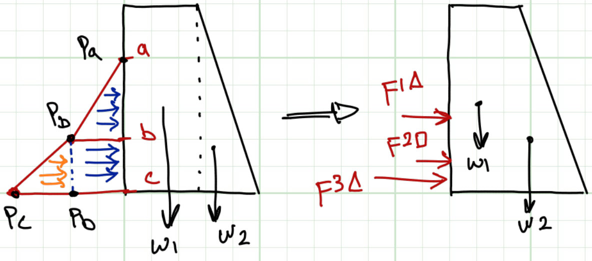
1) FBD ⟨ loads (hydro), weights, supports

2) Calc Pressure ⟷ Forces

3) Calc M_o, M_s

4) $FS = 1.75 = \frac{M_s}{M_o}$, find x

1. FBD



2. Pressures : a, b, c

$$P_A = \rho_w g h = \gamma_w h_a = \underline{\underline{0}}$$

$$P_b = (\rho_w g) h_b = \gamma_w h_b = 1000(9.81)(4) = \underline{\underline{39.24 \text{ kPa}}}$$

$$P_c = \underbrace{\rho_w g h_b}_{} + \underbrace{\rho_s g(h_c - h_b)}_{} = 1000(9.81)(4) + 1760(9.81)(6-4)$$

$$= \underline{\underline{73.77 \text{ kPa}}}$$

water

constant H₂O

silt

(2m)

2. **Pressure ⟷ Forces**

assumed in/out pg.

$$F^{1\Delta} = \frac{1}{2}(39.24)(4)(1) = \underline{\underline{78.48\text{ kN}}}$$

$$F^{2\square} = l \cdot w \cdot h = (39.24)(2)(1) = \underline{\underline{78.48\text{ kN}}}$$

$$F^{3\Delta} = \frac{1}{2} \cdot l \cdot w \cdot h = \frac{1}{2}(73.77 - 39.24)(2)(1) = \underline{\underline{39.53\text{ kN}}}$$

Moment Arms

F_1 acts : $y_{F_1} = 3.33\text{ m}$ from A

F_2 acts : $y_{F_2} = \frac{2}{2}\text{m} = 1\text{ m}$ from A

F_3 acts : $y_{F_3} = \frac{2}{3}\text{m} = 0.666\text{ m}$ from A

Concrete weights — **Moment Arm**

$$W_1^{\square} = 2400(9.81)\overbrace{(9)\left(\frac{x}{2}\right)}^{\text{area }\square} = 105.9x \qquad \bullet\ x - \frac{x}{2}\Big/2 = \frac{3}{4}x$$

$$W_2^{\Delta} = 2400(9.81)\left(\frac{1}{2}\left(x - \frac{x}{2}\right)\right)(9) = 52.97x \qquad \bullet\ x - \frac{x}{2} - \frac{1}{3}\left(x - \frac{x}{2}\right) = \frac{x}{3}$$

Overturning Forces → Mo

$$+\circlearrowleft \Sigma M_A = M_o = F_1^{\Delta}(3.33) + F_2^{\square}(1) + F_3^{\Delta}(0.66)$$

$$\boxed{M_o = 362\text{ kN}\cdot\text{m}}$$

Stabilizing Forces → Ms

$$+\circlearrowleft \Sigma M_A = M_s = -W\left(\frac{3}{4}x\right) - W_2\left(\frac{x}{3}\right) \qquad \boxed{M_s = -97.08x^2}$$

F.S Equation

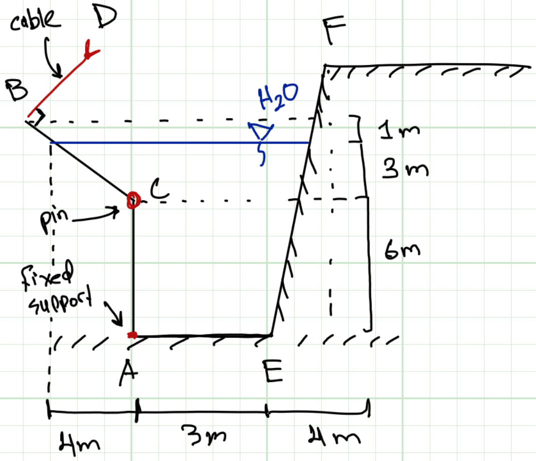
base dimension

$$F.S = \left|\frac{M_s}{M_o}\right| \qquad 1.75 = \frac{97.08x^2}{362\text{ kN}\cdot\text{m}} \Longrightarrow \boxed{x = 2.25\text{ m}}$$

Week 11: Lec #3

Exam Level Hydro Problem

Eg 1:



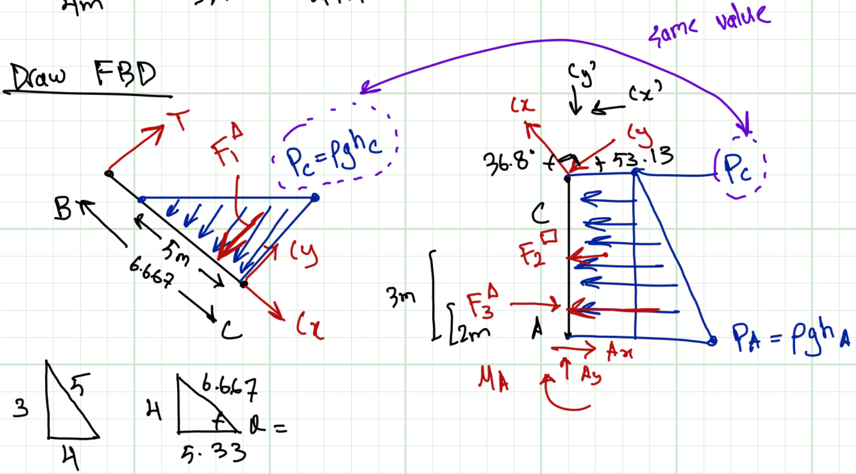
Given

- $\rho_{saltwater} = 1050$ kg/m³
- 8 metres wide, BUT

$\boxed{\text{find A per metre width of wall}}$

4 unknowns: A_x, A_y, M_A, T_{BD}

1. Draw FBD



$$P_C = \rho g h_c = \frac{(1050)(9.81)(3)}{1000} = 30.9 \text{ kPa}$$

$$P_A = \rho g h_A = \frac{(1050)(9.81)(6+3)}{1000} = 92.7 \text{ kPa}$$

$$F_1^\Delta = \frac{1}{2}\cdot b \cdot h = \frac{1}{2}(5)(30.9)\underline{(1m)} = 77.2 \text{ kN/m}$$

$F_2^D = 6 \times 30.9 \times \underline{1} = 185.4$ kN/m

$F_3^A = \frac{1}{2}(6)(1)(P_A - P_c) = 185.4$ kN/m

<u>Apply Eq. Equ.</u>

FBD I $+\circlearrowright \sum M_c = 0 \rightarrow T$

$+\circlearrowright \sum M_B = 0 \rightarrow C_y$ $\quad +\circlearrowright \sum M_B = 0 = 77.18(5) - C_y(6.667)$

$\xrightarrow{+} \sum F_x = 0 = C_x \quad \underline{\underline{C_x = 0}}$ $\quad \underline{\underline{C_y = 57.9 \text{ kN}}}$

FBD II

$C_y \cos 53.13 = C'_x = 34.72$ kN

$C_y \sin 53.13 = C'_y = 46.3$ kN

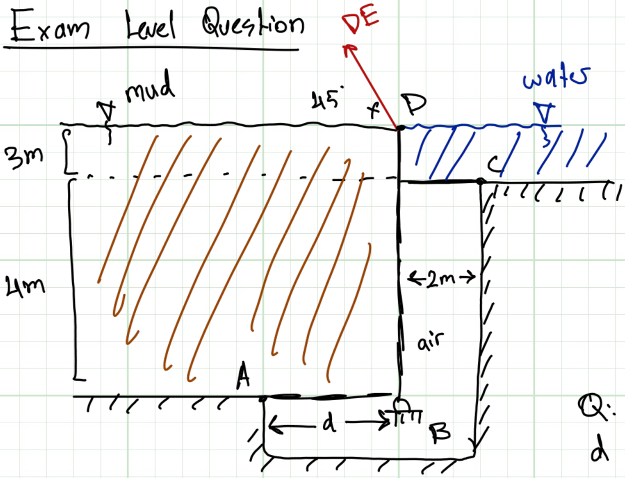
$\xrightarrow{+} \sum F_x = 0 = A_x - 34.72 - F_2^D - F_3^A \quad \boxed{A_x = 405 \text{ kN/m}}$

$+\uparrow \sum F_y = 0 = A_y - 46.3$ kN $\quad \boxed{A_y = 46.3 \text{ kN}}$

$+\circlearrowright \sum M_A = 0 = -F_2^D(3) - F_3^A(2) - C_x'(6) + M_A$

$$\boxed{M_A = 1135 \text{ kN}\cdot\text{m/m} \circlearrowright}$$

Exam Level Question



Given

Hinge at D

Cable DE spread at 2m intervals

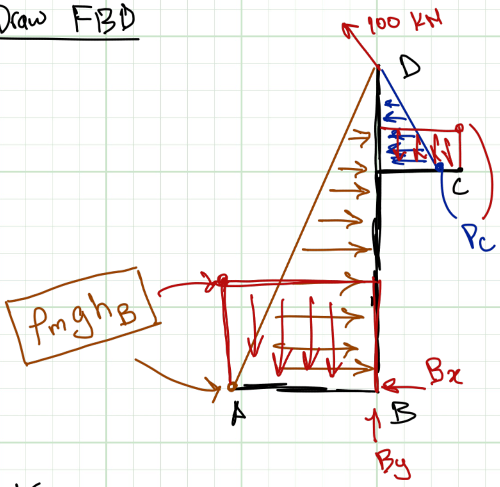
$T_{DE} = 100$ kN

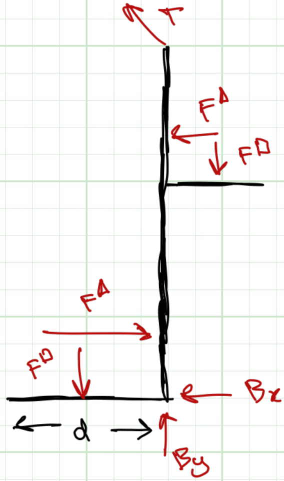
$\rho_w = 1000\ kg/m^3$

$\rho_{mud} = 1300\ kg/m^3$

Q: what is min. distance d for equilibrium

Draw FBD





3 unknowns: d, B_y, B_x

$$P_c^{water} = 29.43\ kPa \quad = \rho_w g h_c$$

$$P_m = \rho_m g h_B \;= 89.27$$

$$F_i^A = 29.43(2)\overset{\text{spacing}}{(2)} \;= 117.72\ kN \text{ at } 1m \text{ from } B$$

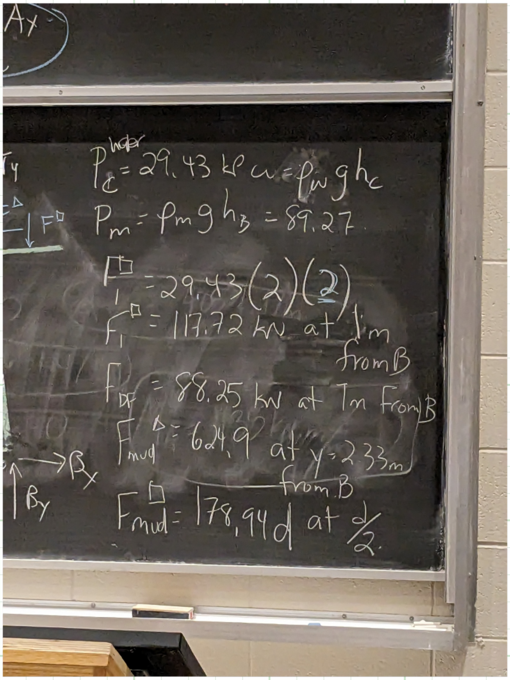
$$F_{DF} = 88.25\ kN \text{ at } 1m \text{ from } B$$

F_{mud}

$+\circlearrowright \sum M_B = 0 =$ cable $+$ water$^{\Delta}$ $-$ water$^{\square}$ $-$ mud$^{\square}$ $-$ mud$^{\Delta}$

$$d^2 = 7.189$$

$$\boxed{d = 2.68m}$$



$P_C^{water} = 29.43$ kP $w = \rho_w g h_c$

$P_m = \rho_m g h_B = 89.27$

$F_1^{\square} = 29.43(2)(2)$

$F_1^{\square} = 117.72$ kN at 1m from B

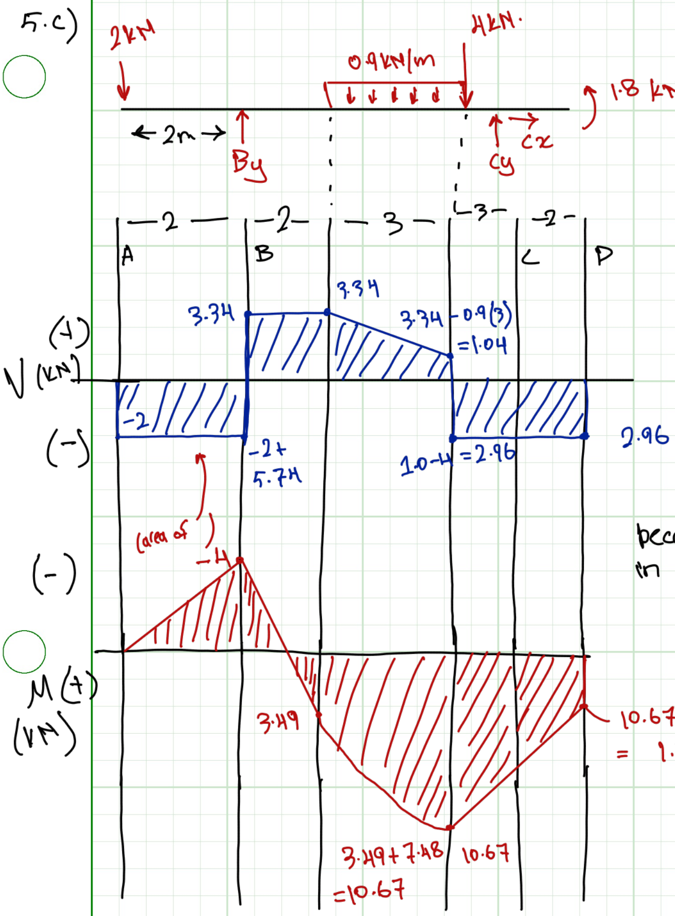
$F_{DF} = 88.25$ kN at 1m from B

$F_{mud}^{\Delta} = 624.9$ at $y = 2.33m$ from B

$F_{mud}^{\square} = 178.94d$ at $d/2$.

Exam Review 1

5.c)



$$+\circlearrowleft \sum M_C = 0$$

$$\underline{\underline{B_y = 5.74 \text{ kN}}}$$

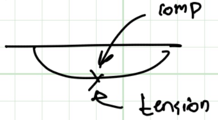
$$\uparrow^+ \sum F_y = 0$$

$$\underline{\underline{C_y = 2.96 \text{ kN}}}$$

$$\rightarrow^+ \sum F_x = 0$$

$$\underline{\underline{C_x = 0}}$$

Option B is better because there is more tension in the bottom



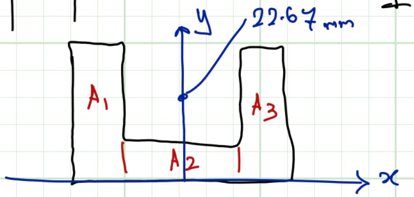
$$\sigma = \frac{M\overset{+ve}{y}}{I} \quad \text{max tens}$$

$$\sigma = \frac{M\overset{-ve}{y}}{I} \quad \text{max com}$$

c) $$\sigma = \frac{(\overset{1.2}{L.F})(\overset{10}{M})y}{I}$$

centroid

I ← moment of inertia



$$\bar{y} = \frac{A_1\tilde{y} + A_2\tilde{y} + A_3\tilde{y}}{A_1 + A_2 + A_3} = 22.65\text{ mm}$$

from bottom

$$I_x = \left[\frac{1}{12}bh^3 + Ad^2\right]^{A1} + \left[\frac{1}{12}bh^3 + Ad^2\right]^{A2} + \left[\frac{1}{12}bh^3 + Ad^2\right]^{A3}$$

$= 30 - 22.65$

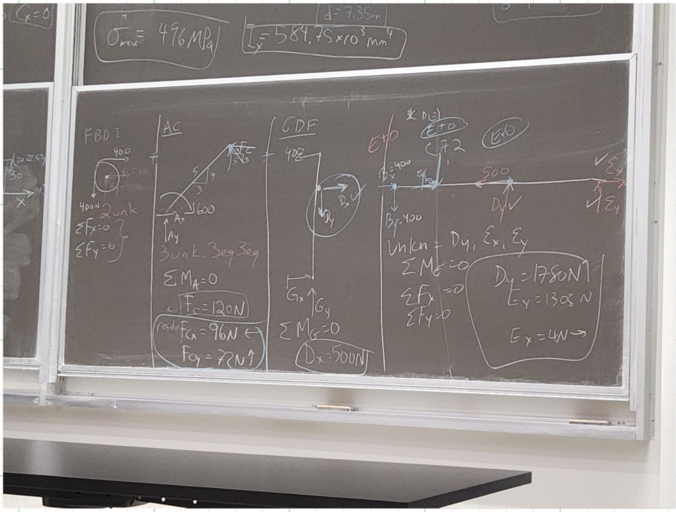
$d = 7.35$

$= 22.65 - 5$

$d = 17.65$ mm

$I_x = 584.75(10)^3 \text{ mm}^4$

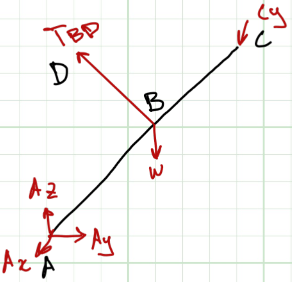
$$\sigma^{max}_{tension} = \frac{1.2(10.67)(22.65)}{584.75(10)^3} = \boxed{496 \text{ MPa} = \sigma_{max}}$$



Week 12:

Question 1 … exclude ball & socket D by cutting BD before D.

* unknowns: $A_x, A_y, A_z, T_{BD}, C_x$



$$\Sigma M_{A_x} = \Sigma M_{A_y} = \Sigma M_{Az} = 0$$

$$\Sigma F_x = \Sigma F_y = \Sigma F_y = 0$$

A(120,0,0)
B(60,30,20)
C(0,60,40)
D(60,0,60)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_i & r_j & r_k \\ F_i & F_j & F_k \end{vmatrix}$$

* Force $\vec{T}_{BD} = \vec{u}_{BD} |T_{BD}|$

$$\vec{u}_{BD} = \frac{\vec{r}_{B-D}}{|\vec{r}_{B-D}|} = \frac{(60-60)\hat{i} + (0-30)\hat{j} + (60-20)\hat{k}}{\sqrt{(-30)^2 + (40)^2}}$$

$$\vec{u}_{BD} = 0\hat{i} - 0.6\hat{j} + 0.8\hat{k}$$

$$\underline{\underline{\vec{T}_{BD} = 0\hat{i} - 0.6 T_{BD}\hat{j} + 0.8 T_{BD}\hat{k}}}$$

* Weight $\vec{w} = -250\hat{k}$

* Support $\vec{C} = F_c\hat{j}$

Moment Arms

- $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$
  $= -60\hat{i} + 30\hat{j} + 20\hat{k}$
- $\vec{r}_{AC} = \vec{r}_C - \vec{r}_A$
  $= -120\hat{i} + 60\hat{j} + 40\hat{k}$

$\Sigma M_A = 0$

$$0 = (\vec{r}_{AB} \times \vec{T}_{BD}) + (\vec{r}_{AB} \times \vec{w}) + (\vec{r}_{AC} \times \vec{F}_c)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -60 & 30 & 20 \\ 0 & -0.6T_{BD} & 0.8T_{BD} \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -60 & 30 & 20 \\ 0 & 0 & -250 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -120 & 60 & 40 \\ F_c & 0 & 0 \end{vmatrix}$$

$$0 = (36T_{BD} - 7500)\hat{i} + (48T_{BD} - 15000 + 40F_c)\hat{j} + (36T_{BD} - 60F_c)\hat{k}$$

$\sum M_{Ax} = 0 = 36T_{BD} - 7500$ $\boxed{T_{BD} = 208}$

$\sum M_{Az} = 0 = 36T_{BD} - 60F_c$ $\boxed{F_c = 120}$

$\sum F_x = 0 = A_x + 125$ $\boxed{A_x = -125}$

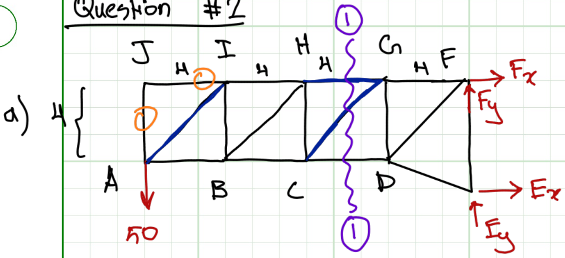
$\sum F_y = 0 = A_y - 0.6(208.33)$ $\boxed{A_y = 125}$

$\sum F_z = 0 = A_z - \underset{w}{250} + 0.8(208.33)$ $\boxed{A_z = 83.3}$

KN

Question #2

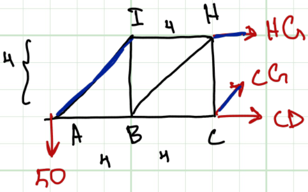
a)



I) Global Equ. → 4prt

II) Internal Equ.

FBD I



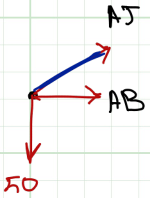
$+\circlearrowleft \sum M_C = 0 = HG(4) - 50(8)$

$\boxed{HG = 1000 \text{ KN (T)}}$

$+\uparrow \sum F_y = 0 = -50 + CG \sin 45°$

$\boxed{CG = 70.7 \text{ KN (T)}}$

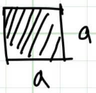
Method of Joints for Joint A



$$+\uparrow \sum F_y = 0 = AJ \sin 45^\circ - 50$$

$$\boxed{AJ = 70.7\ kN\ (T)}$$

b)



$= 24\ MPa$, $LF = 1.2$, $a = ?$

$$\boxed{\sigma = \frac{M(L.F.)\,y}{I} \Longleftrightarrow \sigma = \frac{F(L.F.)}{A}}$$

$$\sigma = \frac{F(L.F.)}{A} \qquad F = 70.7\ kN$$

$$24(10)^6 = \frac{(70.7 \times 10^3)(1.2)}{a^2}$$

$$\boxed{a = 59.5\ mm}$$

(60 x 60)mm cross-section

in Practice